Pandemic-Era Inflation Drivers and Global Spillovers

Julian di Giovanni

Federal Reserve Bank of New York and CEPR Şebnem Kalemli-Özcan

Brown University, CEPR and NBER

Alvaro Silva

Federal Reserve Bank of Boston

Muhammed A. Yıldırım

Harvard & Koç University

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This paper

What?

- * Quantify inflation drivers when economy is hit by different set of shocks
- * Key: open economy \rightarrow allow to study spillovers across countries-sectors

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Why?

- * Highest inflation of last four decades
- * Characterized by:
 - + Collapse and rebound in domestic demand, GDP and international trade
 - + Consumption substitution across sectors
 - + Labor shortages
- * Requires a model

This paper

$\blacktriangleright \ How? \rightarrow {\sf Multi-country\ multi-sector\ model}$

- * Theory: extends Baqaee and Farhi (2022, AER) to an open economy
- * Segmented factor markets + downward nominal wage rigidity
- * Allow for a nominal block to tie real shocks to nominal variables
 - + countries conduct independent monetary policy \rightarrow flexible exchange rates
- * Takes model to the data
 - + 4 Countries: United States, Euro Area, Russia, and China+RoW
 - + 4 Sectors: durables, non-durables, services, and energy (extension to 44 in the paper)
 - + Rich set of shocks: sectoral supply, sectoral demand, aggregate demand and energy shocks
 - + Conduct counterfactuals: what if only supply shocks? ... demand shocks? ... energy shocks?

- * Negative supply shocks to factors of production: \uparrow inflation
- * Negative aggregate demand shocks: \downarrow inflation
- * Low inflation in 2020

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- * Still strong aggregate demand: \uparrow inflation
- * Sectoral supply shocks started to recede: \downarrow inflation
- * Energy price changes
 - + Higher impact on the euro area but increases inflation everywhere: \uparrow inflation

What we find cted'

- International spillovers: larger in euro area than the US.
- Model matches well
 - * other aggregate moments: current account to GDP ratio, bilateral exchange rates (qual.)
 - * cross-sectional moments: sector-level prices and wages
- ► Role of disaggregation: model with more sectors does better

Related literature

Inflation with sectoral demand and supply shocks: theory and empirics

Closed economy

Baqaee and Farhi (2022), La'O and Tahbaz-Salehi (2022), Rubbo (2022), Afrouzi and Bhattarai (2022), Pasten et al (2020), Ferrante et al (2023), Guerrieri et. al (2021, 2022), Lorenzoni and Werning (2023), Blanchard and Bernanke (2023), Gagliardone and Gertler (2023), Benigno and Eggertson (2023), Harding et al (2023), Fornaro and Wolf (2023), Jorda et al (2022), LaBelle and Santacreu (2022), Shapiro (2022), de Soyres et al. (2024), Bai et al (2023)

Open economy

di Giovanni et al (2021), Amiti et al (2022), Silva (2023), Comin et al (2023), Andrade et al (2023), Cuba-Borda et al (2024)

• Our contribution: a structural GE model to quantify inflation drivers with

- * global input-output linkages
- * downward nominal wage rigidity + segmented factor markets
- * endogenous exchange rates
- * rich set of shocks and counterfactuals

Outline

- Model
- Calibration
- Results
- Conclusion



Inflation in a multi-country multi-sector model

- Open economy extension of Baqaee and Farhi (2022, AER):
 - * Two-period multi-country model (n, m = 1, ..., N)
 - * Multiple sectors (i, j = 1, ..., J) produce using factors and intermediate inputs
 - * Ricardian households with perfect foresight
 - * Have access to a domestic and a world bond denominated in US dollars
 - * Perfect competition in factors and good markets
 - * Monetary policy is conducted independently in each country
- Frictions:
 - * Downward nominal wage rigidity (in local currency)
 - * Segmented factor markets

Households in Country n: Inter-temporal Problem

$$\max_{\{C_{n,o}, C_{n,1}, F_{n,o}, B_{n,o}\}} (1 - \beta_{n,o}) \frac{C_{n,o}^{1-\sigma}}{1-\sigma} + \beta_{n,o} \frac{C_{n,1}^{1-\sigma}}{1-\sigma}$$

s.t.
$$P_{n,o}C_{n,o} + B_{n,o} + \mathcal{E}_{n,o}F_{n,o} \le \sum_{i} (W_{ni,o}L_{ni,o} + R_{ni,o}K_{ni,o}),$$
$$P_{n,1}C_{n,1} \le \mathcal{E}_{n,1} \sum_{i} (W_{ni,1}L_{ni,1} + R_{ni,1}K_{ni,1}) + (1 + i_{n,o})B_{n,o} + \mathcal{E}_{n,1}(1 + i_{US,o})F_{n,o},$$

- \triangleright *B_n*: domestic bond denominated in local currency. Traded domestically.
- ► *F_n*: world bond denominated in US dollars. Internationally traded.
- \triangleright \mathcal{E}_n : exchange rate between country *n* and the US (lcu per dollar)
- ► *i*_n: domestic interest rate
- ► *i*_{US}: US interest rate

Households in Country "n": Intertemporal Optimality

Optimality conditions

$$\phi_{n,0} \frac{C_{n,0}^{-\sigma}}{P_{n,0}} = \frac{C_{n,1}^{-\sigma}(1+i_{n,0})}{P_{n,1}}$$
$$(1+i_{n,0}) = (1+i_{US,0})\frac{\mathcal{E}_{n,1}}{\mathcal{E}_{n,0}}$$

(Euler Equation)

(Interest Parity Condition)

- $\phi_{n,o} = (1 \beta_{n,o})/\beta_{n,o}$: country-level aggregate demand shifter
- ► X: steady-state value. O present where shocks happen, 1 future.
- $\hat{X}_t = X_t / X$: deviation from steady-state.
- From now on, assume future variables are at steady state and $\sigma = 1$.

Monetary policy and exchange rates

► World expenditure (in US dollars) is endogenous

$$\widehat{E}_{W,o}^{\$} = \frac{1}{(1+i_{US,o})} \sum_{n} \alpha_{n} \widehat{\phi}_{n,o}; \qquad \alpha_{n} = (P_{n}C_{n}/\mathcal{E}_{n}) \bigg/ \sum_{m} P_{m}C_{m}/\mathcal{E}_{m}$$

Bilateral exchange rates depend only on stance of domestic monetary policies

$$\frac{\mathcal{E}_{n,o}}{\bar{\mathcal{E}}_n} = \frac{(1+i_{US,o})}{(1+i_{n,o})}$$

• We use data on E_n and $1 + i_{n,0}$ to back out discount factor changes

$$\hat{\phi}_{n,o} = \widehat{E}_{n,o}(1+i_{n,o})$$

Disaggregated Consumption

Country-level consumption bundle C_n

- * Aggregates country-specific sectoral consumption bundles C_{n,j}
- * Cobb-Douglas aggregator: $\sum_{j} \Omega_{n,j}^{C} = 1$
- * Introduce country-sector demand shifters via changes in $\Omega_{n,j}^{\mathsf{C}}$ with $\sum_{i} \mathrm{d}\Omega_{n,j}^{\mathsf{C}} = \mathbf{0}$
- ► Country-specific sectoral consumption bundles: C_{n,j}
 - * Aggregates $C_{n,mj}$ across countries m with elasticity ξ^{c}

Disaggregated Production

- Nested CES structure
- Gross output: Y_{ni}
 - * Combines value added bundle (VA_{ni}) and intermediate bundle (Z_{ni}) with elasticity θ
 - * Country-sector productivity A_{ni}
- Value added bundle: VA_{ni}
 - * Combines sector-specific labor (L_{ni}) and sector-specific capital (K_{ni}) with elasticity η
- Intermediate input bundle: Z_{ni}
 - * Combines sector specific sectoral bundles X_{ni,j} with elasticity ε
- Country-specific sectoral bundles: X_{n,j}
 - * Combines country-specific bundles $X_{n,mj}$ across countries (m) with elasticity ξ^s

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 for each country-sector n, i

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Segmented labor markets with downward nominal wage rigidities

$$W_{ni}^{\$} \ge \frac{\overline{W}_{ni}}{\mathcal{E}_{n}}, \quad \overline{L}_{ni} \ge L_{ni}, \qquad \left(\overline{L}_{ni} - L_{ni}\right) \left(W_{ni}^{\$} - \frac{\overline{W}_{ni}}{\mathcal{E}_{n}}\right) = 0 \quad \text{for each country-sector } n, i$$

* \bar{L}_{ni} : country-sector potential labor supply shocks

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• Segmented capital markets: *no* price rigidities $\rightarrow K_{ni} = \bar{K}_{ni}$

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* \bar{L}_{ni} : country-sector potential labor supply shocks

- Segmented capital markets: *no* price rigidities $\rightarrow K_{ni} = \bar{K}_{ni}$
- Asset markets clear: $\sum_{n} F_{n,t} = 0$, $B_{n,t} = 0$

Calibration

Parameters

- Calibrate the model with ICIO 2018 Table from OECD
 - * 4 sectors: durables, non-durables, services, and energy
 - * 4 countries: United States, Euro Area, Russia, and China+RoW
 - * Final use shares
 - * Input shares
 - * Value added shares
 - * Expenditure shares
 - * Allow for initial trade imbalances

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 - * Allow for initial trade imbalances
- Elasticities: p-complementarities
 - * Between value added and intermediate inputs: $\theta = 0.6$
 - * Between labor and capital: $\eta = 0.6$
 - * Among intermediates: $\varepsilon = 0.2$
 - * Cross-country Armington: $\xi^{s} = \xi^{c} = 0.6$
 - * more configurations in the paper

(Atalay, 2017; Carvalho et. al, 2021)

(Raval, 2019; Oberfield and Raval, 2021)

(Atalay, 2017; Boehm, Flaaen, and Pandalai-Nayar, 2019)

(Boehm, Levchenko and Pandalai-Nayar, 2023)

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 $i \in \mathcal{J}$

- 3. Country-sector potential supply shocks: $\hat{L}_{ni,o}$
 - + Observed changes in total hours worked in country *n*, sector *i*

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 - 4. Energy shocks: IMF commodity price index
 - + maps to productivity shock in Russian energy sector in 2022 $\hat{A}_{(Russia, Energy),o}$ (lower bound)

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 - 4. Energy shocks: IMF commodity price index
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 - 5. Shutdown productivity changes for all other country-sector pairs: $\hat{A}_{ni,o} = 1$

Aggregate results

Model with all shocks: headline inflation



Full model can replicate most inflation pattern in both countries

Euro-USD exchange rate changes



Model predict qualitatively similar behavior to nominal exchange rate in the data

Current account



Reasonable current account movements



Sectoral supply inflationary in 2020 – 2021



Not much role for sectoral demand changes



Aggregate demand deflationary in 2020, inflationary thereafter



Energy shocks important for Euro Area

Domestic and international shocks



Domestic shocks were the predominant force.

Domestic and international shocks



Important International spillovers, especially in the Euro Area.

International demand account for international spillovers



International demand account for most of international spillover in the US.

International demand account for international spillovers



International demand/energy account for the international spillover in the Euro Area.

Cross-sectional results

Sectoral prices



Able to quantitatively match cross-sectional price changes.

Does more disaggregation give us anything?

YES!



Conclusion

Conclusion

- ▶ We develop a model with IO linkages, nominal rigidities and trade across countries
- We use it to study the recent inflationary episode 2020–2022
- Model can match aggregate and sectoral outcomes
- Provided a set of shocks \rightarrow useful for policy makers
- ► Takeaway: more about supply-demand imbalances than supply/demand alone.
- Much more work to do!

Thank you!

asilvub.github.io

asilvub@gmail.com

Calculating Inflation – Auxiliary Matrices

Industry shares in consumption baskets:

 $\Omega^{\text{CS}}\equiv\Omega^{\text{C}}\Omega^{\text{CB}}.$

Industry to industry flows:

 $\Omega^{SS} \equiv \Omega^{Y} \Omega^{Z} \Omega^{X}.$

• All direct and indirect flows from industry to industry (Leontief Inverse): $\Psi = \left[I - \Omega^{SS}\right]^{-1}$

Factor shares (for all factors, including labor and capital):

 $\Omega^{\mathsf{F}} \equiv \Omega^{\mathsf{Y}} \Omega^{\mathsf{V}\mathsf{A}}.$

1. Prices in dollars $(d \log P^{\$})$:

 $\mathrm{d}\log \textit{P}^{\$} = -\Psi \mathrm{d}\log\textit{A} + \Psi \Omega^{\textit{F}} \mathrm{d}\log\textit{W}^{\$}$

2. Country's *n* CPI changes

$$\mathrm{d}\log CPI_n = (\Omega_n^{\mathrm{CS}})^{\mathrm{T}} \mathrm{d}\log P^{\mathrm{LC},n} = \mathrm{d}\log \mathcal{E}_n + (\Omega_n^{\mathrm{CS}})^{\mathrm{T}} \mathrm{d}\log P^{\$}$$

3. Relate factor price f to its factor share at the world level $\Lambda_f = W_f^{\$} L_f / E_W^{\$}$

$$d \log W_f^{\$} = d \log E_W^{\$} + d \log \Lambda_f - d \log L_f$$

• CPI changes (where $(\lambda^n)^T = (\Omega_n^{CS})^T \Psi$ and $(\Lambda^n)^T = (\lambda^n)^T \Omega^F$)

$$d \log \mathsf{CPI}_n = \underbrace{d \log E_W^{\$}}_{\text{World Expenditure}} + \underbrace{d \log \mathcal{E}_n}_{\text{Exchange Rate}} - \underbrace{(\lambda^n)^T d \log A}_{\text{Productivity Shocks}} - \underbrace{(\Lambda^n)^T d \log L}_{\text{Factor Changes}} + \underbrace{(\Lambda^n)^T d \log \Lambda}_{\text{D-S Imbalance}}$$

World expenditure: US interest rate and countries intertemporal shifters.

• CPI changes (where $(\lambda^n)^T = (\Omega_n^{CS})^T \Psi$ and $(\Lambda^n)^T = (\lambda^n)^T \Omega^F$)

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Exchange rate term: country interest rate relative to the US.

• CPI changes (where $(\lambda^n)^T = (\Omega_n^{CS})^T \Psi$ and $(\Lambda^n)^T = (\lambda^n)^T \Omega^F$)

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$$- \underbrace{(\Lambda^n)^T d \log L}_{\text{Factor Changes}} + \underbrace{(\Lambda^n)^T d \log \Lambda}_{\text{D-S Imbalance}}$$

Productivity shocks: Productivity changes weighted by the importance of sector in consumption basket of country n.

• CPI changes (where $(\lambda^n)^T = (\Omega_n^{CS})^T \Psi$ and $(\Lambda^n)^T = (\lambda^n)^T \Omega^F$)



Factor quantity changes: Labor changes weighted by the importance of factor in providing for the consumption basket of country n.

* An endogenous object due to downward-wage rigidity.

• CPI changes (where $(\lambda^n)^T = (\Omega_n^{CS})^T \Psi$ and $(\Lambda^n)^T = (\lambda^n)^T \Omega^F$)



• Changes in global factor shares and local factor shares.

- * Endogenous object: integrates changes in demand and supply factors.
- * Depends on global IO structure and substitution patterns.

Local-Global Demand-Supply Imbalance



Aggregate real wages



Real wages: cross-section



Disaggregated Consumption

Consumption bundle consists of country-specific sectoral consumption bundles:

$$C_n = \prod_{j=1}^{\mathcal{J}} C_{n,j}^{\Omega_{n,j}^c}, \quad \sum_{j=1}^{\mathcal{J}} \Omega_{n,j}^c = 1$$

Country-specific sectoral consumption bundles: Armington aggregator

$$C_{n,j} = \left[\sum_{m=1}^{\mathcal{C}} (\Omega_{n,mj}^{CB})^{\frac{1}{\xi^{c}}} C_{n,mj}^{\frac{\xi^{c}}{\xi^{c}}-1}\right]^{\frac{\xi^{c}}{\xi^{c}-1}}, \quad \sum_{m=1}^{\mathcal{N}} \Omega_{n,mj}^{CB} = 1$$

Disaggregated Production

Sectors produce combining factors (value-added) and intermediate bundle.

$$\begin{split} \min_{\{\forall A_{ni}, M_{ni}\}} P_{ni}^{\forall A} \forall A_{ni} + P_{ni}^{M} Z_{ni} \\ \text{s.t.} \\ Y_{ni} = A_{ni} \left[(\Omega_{ni, \forall A}^{\gamma})^{\frac{1}{\theta}} \forall A_{ni}^{\frac{\theta-1}{\theta}} + (\Omega_{ni, Z}^{\gamma})^{\frac{1}{\theta}} Z_{ni}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad \text{with} \quad \Omega_{ni, \forall A}^{\gamma} + \Omega_{ni, Z}^{\gamma} = 1 \end{split}$$

Value-added bundle is composed of Labor and Capital:

$$\mathsf{VA}_{ni} = \left[(\Omega_{ni,L}^{\mathsf{VA}})^{\frac{1}{\eta}} (L_{ni})^{\frac{\eta-1}{\eta}} + (\Omega_{ni,K}^{\mathsf{VA}})^{\frac{1}{\eta}} (\bar{K}_{ni})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad \text{with} \quad \Omega_{ni,L}^{\mathsf{VA}} + \Omega_{ni,K}^{\mathsf{VA}} = 1$$

Intermediate goods' aggregation

Intermediate bundle consists of country specific sectoral bundles:

$$Z_{ni} = \left[\sum_{j=1}^{\mathcal{J}} (\Omega_{ni,j}^{Z})^{\frac{1}{\varepsilon}} X_{ni,j}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{with} \quad \sum_{j=1}^{\mathcal{J}} \Omega_{ni,j}^{Z} = 1$$

Country-specific sectoral bundles: Armington aggregator

$$X_{n,j} = \left[\sum_{m=1}^{\mathcal{N}} (\Omega_{n,mj}^{X})^{\frac{1}{\xi_{j}^{S}}} X_{n,mj}^{\frac{\xi_{j}^{S}-1}{\xi_{j}^{S}}}\right]^{\frac{\xi_{j}^{S}}{\xi_{j}^{S}-1}} \quad \text{with} \quad \sum_{m=1}^{\mathcal{N}} \Omega_{n,mj}^{X} = 1$$

 $X_{n,j} = \sum_{i} X_{ni,j}$ for *i*'s within country *n*.