Pandemic-Era Inflation Drivers and Global Spillovers*

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Abstract

We estimate a multi-country multi-sector New Keynesian model to quantify the drivers of domestic inflation during 2020–2023 in several countries, including the United States. The model matches observed inflation together with sector-level prices and wages. We further measure the relative importance of different types of shocks on inflation across countries over time. The key mechanism, the international transmission of demand, supply and energy shocks through global linkages helps us to match the behavior of the USD/Euro exchange rate. The quantification exercise yields four key findings. First, negative supply shocks to factors of production, labor and intermediate inputs, initially sparked inflation in 2020–2021. Global supply chains and complementarities in production played an amplification role in this initial phase. Second, positive aggregate demand shocks, due to stimulative policies, widened demand-supply imbalances, amplifying inflation further during 2021–2022. Third, the reallocation of consumption between goods and service sectors, a relative sector-level demand shock, played a role in transmitting these imbalances across countries through the global trade and production network. Fourth, global energy shocks have differential impacts on the US relative to other countries' inflation rates. Further, complementarities between energy and other inputs to production play a particularly important role in the quantitative impact of these shocks on inflation.

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"At the outset, many forecasters and analysts ... viewed the sudden upturn in inflation as mostly a function of pandemic-related shifts in the composition of demand, a disruption of supply chains, and a sharp decline in labor supply. The resulting supply and demand imbalances led to large increases in the prices of a range of items most directly affected by the pandemic, especially goods ... But in the fourth quarter of 2021, the data clearly changed ... with only gradual progress in restoring global supply chains, and relatively few workers rejoining the labor force ... A new shock arrived in February 2022, when Russia invaded Ukraine, resulting in a sharp increase in energy and other commodity prices ... it was clear that bringing down inflation would depend both on the unwinding of the unprecedented pandemic-related demand and supply distortions and on our tightening of monetary policy, which would slow the growth of aggregate demand, allowing supply time to catch up."

Remarks from Jay Powell, 24th Jacques Polak Annual Research Conference, IMF, 2023.

1 Introduction

During the last three years, advanced countries have experienced inflation rates not seen in four decades. The underlying causes of this inflationary episode are still debated. This paper develops a New Keynesian open-economy model to quantify the key drivers of aggregate inflation across several countries, notably the United States and the Euro Area. Our quantification exercise based on the model matches observed aggregate inflation experiences along with sector-level price and wage changes for durables, non-durables, services, and the energy sector.

The model allows for a multitude of shocks at the sector and aggregate levels. Accounting for micro supply and demand shocks has important implications for the employment/output-inflation trade-off both at the sector- and aggregate levels, since this trade-off can be worse when negative supply shocks and positive demand shocks coincide. In this sense the framework in our paper is similar to those developed in the *closed-economy* papers of Baqaee and Farhi (2022) and Rubbo (2023a). By extending the model to the open economy, we highlight important features in the transmission of shocks across the world that not only help us match the observed inflation rates in several countries along with exchange rate movements, but also allows us to quantify the impact of shocks stemming from China lockdown policies and the Russia-Ukraine War on prices and wages in the US and Euro area.

The estimation of our global structural macroeconomic network model allows us to quantify the different forces that drove the pandemic-era inflation at different times in different countries during the last three years, while showing the importance of interconnections across sectors and countries. We quantify the following narrative of three distinct phases in the rise of global inflation, as also highlighted in our opening quote. In the early phase of 2020–2021, supply shocks arising from pandemic-induced scarcity in factors of production, such as constrained imported intermediates and domestic labor, sparked inflation. This period was characterized by local and global supply chain bottlenecks, rising factor costs including prices of imported intermediate inputs together with slack in domestic labor markets. The initial rise in product prices – also highlighted by Blanchard and Bernanke (2023) and Lorenzoni and Werning (2023) – is an important feature of this episode that our model matches. Further, the model is able to match the observed rise in real wages during early 2020, before they fell in 2021, given the early negative labor supply shock.

There were also large fiscal packages and loose monetary policy during the first phase, especially in advanced countries. The fiscal packages aimed at mitigating the economic hardship of higher unemployment came in different phases and lasted into 2022. The aggregate demand shock resulting from easy policies intensified the original supply chain bottleneck problem by stimulating demand in a low supply world, leading to rising inflation during the second phase over 2021–2022.[1] The final phase (2022–2023) was characterized by the Russian invasion of the Ukraine further intensifying inflation given the impact on energy prices, particularly in Europe.

By extending the *closed-economy* contribution of Baqaee and Farhi (2022) to *multiple countries*, we link the fall in sector-specific labor supply (and other factors of production), domestic or foreign, to aggregate output, wages and inflation in all countries. The model incorporates a global production network, which allows for trade in intermediate goods and complementarities in production.[2] The emphasis on factor markets with inelastic factor supply and factor scarcity is combined with several other ingredients: (i) standard aggregate demand shocks (e.g., fiscal stimuli and easy/tight monetary policy) and downward nominal wage rigidity, (ii) sector-specific demand shocks, and (iii) energy sector-specific TFP shocks, fed through Russian energy sector, which capture energy price shocks due to the war. Our quantitative exercises identify the contribution of each of these shocks to domestic inflation.

The baseline analysis includes several other features that help match the data. First, given the short-run complementarity between factors of production in the data (e.g., Boehm, Levchenko,

[1] As di Giovanni, Kalemli-Özcan, Silva, and Yıldırım (2023) show, the three fiscal package in the US drives 60 percent of the US inflation during 2021–2022. The dates of the US fiscal packages are December 2020, March 2021, December 2021.

[2] Çakmaklı, Demiralp, Kalemli-Özcan, Yeşiltaş, and Yıldırım (2021) also extends Baqaee and Farhi (2022) into multiple countries, focusing on real output losses through production network without nominal rigidities.

and Pandalai-Nayar, 2023), the effect of any input shortage in a given country-sector is amplified to production worldwide in model calibrations that use data-consistent elasticities. This feature is particularly important in terms of the complementarity between oil as an input and domestic labor, as also highlighted by Gagliardone and Gertler (2023). Second, we assume segmented factor markets (labor and capital) where reallocation across sectors is limited, consistent with the data during this period.[3] Third, using information on cross-border ownership of factors we solve for model-consistent exchanges rates and trade balances, which track their observed values in the data.[4] The model is non-linear and needs to be solved computationally. However, like Baqaee and Farhi (2022) our framework allows for an analytical first-order approximation solution for each country's inflation rate, which, differently than Baqaee and Farhi (2022), in an open-economy setting is a function of shocks in all countries and sectors.

The model performs well in matching the observed inflation in the US and the Euro Area during 2020–2023. The quantification exercise is reported for an aggregated four-region-four-sector world, given the limitations in real-time sector-level data from several countries: the US, Euro Area, Russia, and an aggregate of the rest of the world, which includes China and that we refer to as "China+". We collect data on sector-level employment and consumption shares, aggregate expenditures, and energy prices at the quarterly level across countries in order to construct our series of shocks. We combine these data with detailed *pre-pandemic* cross-country input-output tables in order to first solve the model at an initial steady-state and then shock the steady-state each quarter with sector-level and aggregate shocks and solve for prices, wages, rental rates of capital and output for all country-sectors in the world together with exchange rates. Note that the non-linear model solves for all prices and quantities, including model-implied "new" global network (expenditure) shares under shocks.[5] We then compute year-on-year inflation on aggregate prices at each quarter as the log-difference between a given quarter relative to the same quarter in the previous year.

[3]As shown by Fernald and Li (2022), during 2020-2022, the contribution of labor reallocation from low to high wage/productivity sectors is very small, 0.18 percent, whereas labor productivity grew 1.1 percent.

[4]Our approach is in the spirit of Dekle, Eaton, and Kortum (2007, 2008) focusing on capital (factor income) inflows, but different than Baqaee and Farhi (Forthcoming), who accommodate standard exchange rate determination with full wage rigidity for tariff shocks. See also Gourinchas et al. (2021) who solves for changes in current accounts under fixed exchange rates.

[5]Both the OECD and the U.S. BEA data on network input-output shares come with 5-7 year lag and hence actual network expenditure shares in 2020-2021 will not be known until 2025-2027.

The model is solved each quarter by feeding in shocks, defined relative to a pre-pandemic period. We collect the post-shock CPI levels each period and use these to calculate model-based inflation rates between 2020Q1-2022Q4. Thus, while we do not have persistence in the model, we are still able to map out a path of inflation over the time series. To summarize how well model-generated inflation rates compare to the data at the annual level, we calculate quarter-specific year-on-year inflation rates for each quarter in a given year and then take the arithmetic average of these numbers for the given year. The baseline model generates inflation rates of (in percent) 2.30, 8.61, and 6.37 in the United States for 2020, 2021, and 2022, while actual headline inflation during these years was 1.23, 4.98, and 7.72 percent, respectively. Thus, our model predicted-inflation rates broadly match those observed in data on average.[6]

Next, to quantify the sources of inflation, we feed each shock into the model separately to calculate their contribution to inflation. This exercise yields an interesting historical decomposition of the drivers of inflation. For example, for the US in 2020, allowing for only sector-level supply shocks would have generated 2.62 percent inflation (higher than reality); including only sector-level demand shocks would have generated 0.93 percent inflation (lower than reality); including only aggregate demand shocks would have generated -0.28 percent inflation (much lower than reality), and having only energy shocks would have led to -0.32 percent inflation. These findings suggest that supply shocks were more important initially and aggregate demand shocks were disinflationary. In 2021, sector-level supply shocks would have generated an inflation rate of -1.74 percent; only sector-level demand 0.65, only aggregate demand 7.88, and only energy 1.12, suggesting that aggregate demand shocks were more important in 2021 given that actual inflation was approximately 5 percent. During 2022, sector-level supply shocks only would have generated an inflation rate of -1.07 percent, sector-level demand an inflation rate of -0.18 percent, aggregate demand an inflation rate of 7.57 percent, and energy an inflation rate of 1.69 percent. Therefore, actual inflation would have been much higher without the improvement in supply chains that acted as a disinflationary force (measured via the sector-level supply shock) in the last part of 2021 and throughout 2022.

Turning to the results for the Euro Area, the model also performs well in matching the trends

[6] The model overshoots 2021 inflation, but as discussed in Section 4, this is due to base effects impact of the massive rebound arising from the reopening of the economy.

in actual inflation, as well as highlighting the importance of energy shocks for the European experience. The baseline model generates inflation rates of -2.67, 7.27, and 8.13 percent for 2020, 2021, and 2022, respectively. Inflation in the data was 0.11, 2.84, and 8.43 percent over the same period. The overall story of the shocks' contribution is similar to the United States but with some important differences, especially for the role of energy shocks. During 2020, only sector-level supply shocks would have generated 1.59 percent inflation (again much higher than reality), only sector-level demand would have generated 0.11, only aggregate demand -3.10, and only energy -0.56 percent inflation, respectively. These findings suggest that a combination of sector-level supply and aggregate demand shocks can help to rationalize the low inflation during this year. In 2021, the contributions of the different shocks were -0.38, -0.21, 5.06, and 1.9 percent, respectively, suggesting that aggregate demand and energy played important roles in driving Euro Area inflation. Finally, the story in 2022 is similar to that of 2021. In particular, the contributions of sector-level supply, sector-level demand, aggregate demand, and energy shocks were -1.48, -0.18, 8.98, and 2.81 percent, respectively, suggesting again a predominant role for shocks to the energy sector playing an important in explaining Euro Area inflation.

The model also allows us to examine the quantitative importance of complementarities in production. These complementarities play an important role in the response of prices to shocks insofar as these elasticities dictate how much a fall in the supply of goods from one country-sector can be substituted with varieties from other countries. When these elasticities are low, the impact of international shocks become more pronounced.

The baseline quantification exercise also produces exchange rate and current account dynamics that are consistent with the data. We solve for these variables as follows. The model features a downward nominal wage rigidity that is expressed in each country's local currency, while the world equilibrium must be solved in a common currency. To solve the model, we must make one further assumption that households in each economy hold claims on income streams from trading partners, which drives current account movements after shocks hit the economy. With this additional assumption, the model delivers bilateral nominal exchange rates so that markets clear both domestically and internationally at steady-state and after the economies are hit by shocks. While not the main focus of our analysis, it is reassuring that the model-generated USD/euro exchange rate dynamics match movements of the actual USD/euro exchange rate in the data: the correlation between the annual percentage change of the model and data exchange rate is 0.93. This result is somewhat surprising given that the model ignores financial market dynamics, such as "flight to safety" at the onset of the pandemic, that many revert to explain appreciation of the USD early on, which later intensified with the tight monetary policy stance.[7] Our model can match the observed USD appreciation due to the large aggregate demand shock in the US via the fiscal stimulus that decreased savings, widened the current account deficit thus increasing capital inflows to the US and appreciating the USD.[8]

International spillovers operate through several channels in amplifying or mitigating inflation at home in the model. Using a first order approximation, we formally show – see Proposition 1 in Section 2 – that the degree to which supply shocks spillover over from foreign countries to impact domestic inflation depends on domestic consumption's exposure to foreign factors of production. This exposure depends both on the consumption of foreign *final* goods, but also on the degree to which any consumption good embeds foreign factors of production. In particular, final goods are produced using intermediate goods, which may be sourced from multiple countries and pass through several stages of production. Therefore, the degree to which domestic consumption depends on foreign factors in turn depends on network effects arising from global production linkages, which will amplify shocks to foreign factors. This dependency varies greatly from country to country – for example, the US shows relatively less dependency on foreign factors compared to the Euro Area.[9] This mechanism also connects labor market to other factors of production. For example, if there is Keynesian unemployment in the foreign country due to downward wage rigidity, this will be an inflationary cost-push at home as home global factor usage goes down via complementarity. However, if a foreign aggregate demand shock remedies the Keynesian unemployment problem (foreign fiscal stimulus), then inflation goes down at home. Hence, fiscal stimulus in the U.S. might increase inflation in the U.S. but decrease it in the rest of the world, if the U.S. labor market is demand constrained and there is downward nominal wage rigidity. Finally, exchange rates mitigate the impact of the transmission of aggregate demand shocks across countries. In fact,

[7] For example, models that try to capture USD appreciation in bad times work through a convenience yield/safety premium (e.g., Kekre and Lenel, 2023), which we do not have.

[8] We do not have explicit easy monetary policy in 2020-2021 and tight policy in 2022, but we have it implicitly via aggregate expenditures.

[9] Europe's higher energy dependence from Russia vis-a-vis the US is a typical example, however for different factors of production these dependencies might differ across countries and hence the entire network needs to be taken into account for the precise measurement of international spillovers.

if wage rigidity is not binding (will be under positive demand shocks), the cross-country effects of aggregate demand shocks on inflation rates are completely offset by exchange rate adjustments, via the standard Mundell-Fleming channels.

Related Literature

Our paper relates to the rapidly growing literature studying the inflation of 2021–2023. As in closed economy papers of Baqaee and Farhi (2022), Rubbo (2023b), Guerrieri et al. (2021), Lorenzoni and Werning (2023), we emphasize a multi-sector approach to modelling pandemic-era inflation, input-output linkages, production complementarities and downward nominal wage rigidities. As in Gagliardone and Gertler (2023) and Blanchard and Bernanke (2023), we highlight the role of oil shocks, product price increases and labor market tightness. As in Comin, Johnson, and Jones (2023), Amiti, Heise, Karahan, and Şahin (2023) and Ferrante, Graves, and Iacoviello (2023), we emphasize the role of global supply chains. We differ from these papers in that we allow for a full global trade and production network input-output structure, so that all sector and aggregate price changes are endogenous to shocks across country-sectors in the world. These papers instead limit their analysis to two sectors in a small open economy setting and, hence, have to take foreign import prices as exogenous and since their shocks to labor are aggregate, they will miss the interaction between labor market dynamics in services vs good sectors and prices in these sectors. We also relate to studies that emphasize a non-linear Phillips curve, such as Eggertson and Kohn (2023) and Benigno and Eggertsson (2023), that link the slope of the non-linear Phillips curve to labor market slackness and tightness. In contrast to these models, our non-linear sector-level approach can account for variety of shocks explaining inflation at different times in different countries together with the co-existence of slack and tight labor market sectors.

Our paper fits in the literature that focuses on inflation and monetary policy with input-output linkages such as Basu (1995), Erceg et al. (2000), Aoki (2001), Woodford (2003), Blanchard and Gali (2007), La'O and Tahbaz-Salehi (2022), Carvalho (2006), Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008, 2010, 2013), Carvalho and Nechio (2011), Bouakez et al. (2014), Pasten et al. (2020, Forthcoming), Castro (2019), Höynck (2020), Baqaee and Farhi (2022) and Rubbo (2023a). Our difference is open economy and quantification of the pandemic-era inflation. We do not study optimal monetary policy. There is also reduced-form empirical studies that seeks to identify the different drivers of inflation with sign restrictions in VAR such as Jordá, Liu, Nechio, and Rivera-Reyes (2022), Shapiro (2022a,b), and Jordá and Nechio (2022).

The contribution of our work over the existing literature that tries to understand the Covid-era inflation is the ability to quantify the role of four sets of shocks (aggregate and sector-level) to the different phases of inflation over 2020–2023 and study the transmission of these shocks across sectors and countries given our global trade and production network structure.[10] The model's micro-structure is rich enough to study how different assumptions on production and consumption substitutability impact the contribution of different shocks to inflation, both domestically and abroad, which is central to the recent de-globalization debate.

Outline of the Paper

Section 2 outlines the multi-country multi-sector model that we use to quantify the drivers of inflation. Section 3 describes the data and shock construction that we use for the quantification exercises. Section 4 presents the quantitative results. Section 5 concludes.

2 Model

We extend the Baqaee and Farhi (2022) model to an open-economy setting by incorporating cross-country and cross-sector input output linkages, as well as endogenous exchange rate and current account adjustments. The model allows for a rich set of shocks including country-level aggregate demand shocks, country-sector level demand shifts, and country-sector level factor supply and productivity shocks.

Similar to Baqaee and Farhi (2022), we simplify the dynamics of the model with the assumption that all countries are at steady-state levels of production and consumption before and after shock periods. In each shock period, the households and producers expect to return to this steady state. We distinguish the variables related to the steady state with an asterisk ($*$).

We denote countries with indices $m, n = 1, \ldots, \mathcal{N}$, where $\mathcal{N}$ is the number of countries. We use $i, j, k = 1, \ldots, \mathcal{J}$ as sector indices. A sector in a country is identified by a pair of indices corresponding to countries and sectors, respectively.

[10] The policy extension of our paper focusing only on US and Europe in 2021 that was prepared for the 2022 ECB-Sintra conference embeds a fixed-exchange rate regime.

2.1 Households

We assume that households are Ricardian and have perfect foresight. Households in country n makes their consumption decisions by maximizing the following inter-temporal utility:

$$\max_{\{C_n, C_n^*\}} (1-\beta_n)\log C_n + \beta_n \log C_n^*, \tag{1}$$

where C_n and C_n^* are the consumption bundles for the shock period and the future, respectively. The β_n parameter weighs households' time preferences and we assume an inter-temporal elasticity of substitution of 1 (therefore the Cobb-Douglas nature of the inter-temporal utility). While optimizing their consumption decision, households respect their inter-temporal budget constraints:

$$P_n C_n + \frac{P_n^* C_n^*}{1+i_n} = I_n + \frac{I_n^*}{1+i_n}, \tag{2}$$

where P_n (P_n^*) is the price of the consumption bundle and I_n (I_n^*) is nominal income in the current (future) period, and i_n is the nominal interest rate in the shock period.

Households' final consumption bundle is a Cobb-Douglas aggregate over *sector-level* consumption bundles:

$$C_n = \prod_{j=1}^{\mathcal{J}} C_{n,j}^{\Omega_{n,j}^C} \quad \text{with} \ \sum_{j=1}^{\mathcal{J}} \Omega_{n,j}^C = 1,$$

where $C_{n,j}$ denotes the country n's consumption bundle of sector j's goods (or services), and $\Omega_{n,j}^C \geq 0$ represents the household's consumption share of this sector. The sector-level consumption bundles are in turn aggregates of varieties of goods from different countries in a given sector. Let $C_{n,mj}$ denote the consumption of output of sector j in country m by consumers in country n. Then the country n-sector j consumption bundle is formed by the following CES aggregation:

$$C_{n,j} = \left[\sum_{m=1}^{\mathcal{N}} \Omega_{n,mj}^{CB} C_{n,mj}^{\frac{1-\xi_c}{\xi_c}}\right]^{\frac{\xi_c}{1-\xi_c}} \qquad \text{with} \ \sum_{m=1}^{\mathcal{N}} \Omega_{n,mj}^{CB} = 1,$$

where $\Omega_{n,mj}^{CB} \geq 0$ is the weight of country-sector mj in country n's consumption of sector j, and ξ_c captures the elasticity of substitution between these varieties.

2.2 Production

Goods are produced at the sector level by combining different factors of production and intermediate inputs. We assume that factors are sector-specific labor and capital, and to help with notation we combine these to create a *value-added* bundle. Each sector in each country uses goods from other countries to construct their sector specific intermediate bundles.

Sector i in country n, therefore, uses sector specific value-added (VA_{ni}) and intermediate bundle (Z_{ni}) to produce its final good via using the following production function:

$$Y_{ni} = A_{ni}\left[\Omega^{Y}_{ni,\mathrm{VA}}\mathrm{VA}_{ni}^{\frac{1-\theta}{\theta}} + \Omega^{Y}_{ni,Z}Z_{ni}^{\frac{1-\theta}{\theta}}\right]^{\frac{\theta}{1-\theta}} \qquad \text{with } \Omega^{Y}_{ni,\mathrm{VA}} + \Omega^{Y}_{ni,Z} = 1, \tag{3}$$

where Y_{ni} denotes the output of this sector and A_{ni} is the sector-specific productivity parameter. Production follows a CES aggregation where θ determines the elasticity of substitution between the value added and the intermediate bundle. Finally, $\Omega^{Y}_{ni,\mathrm{VA}}$ and $\Omega^{Y}_{ni,Z}$ are the shares of value added and the intermediate good used in the final good's production, respectively.

The value-added bundle for country-sector ni consists of sector-specific labor and capital. We assume that capital is always fully utilized and is always at its steady-state value (K^*_{ni}). Labor levels, on the other hand, may potentially fluctuate from their steady-state value when the economy experiences shocks. The value-added bundle is defined as:

$$\mathrm{VA}_{ni} = \left[\Omega^{\mathrm{VA}}_{ni,L}\left(L_{ni}\right)^{\frac{1-\eta}{\eta}} + \Omega^{\mathrm{VA}}_{ni,K}\left(K^*_{ni}\right)^{\frac{1-\eta}{\eta}}\right]^{\frac{\eta}{1-\eta}} \qquad \text{with } \Omega^{\mathrm{VA}}_{ni,L} + \Omega^{\mathrm{VA}}_{ni,K} = 1,$$

where η is the elasticity of substitution between labor and capital, and $\Omega^{\mathrm{VA}}_{ni,L}$ ($\Omega^{\mathrm{VA}}_{ni,K}$) is the weight of value-added that is attributed to labor (capital).

Similar to consumption bundles, the intermediate bundles are constructed from country-specific sector bundles given the following CES aggregator with an elasticity of substitution of ε:

$$Z_{ni} = \left[\sum_{j=1}^{\mathcal{J}}\Omega^{Z}_{ni,j}X_{ni,j}^{\frac{1-\varepsilon}{\varepsilon}}\right]^{\frac{\varepsilon}{1-\varepsilon}} \qquad \text{with } \sum_{j=1}^{\mathcal{J}}\Omega^{Z}_{ni,j} = 1,$$

where $\Omega^{Z}_{ni,j} \geq 0$ is sector j's weight in producing country-sector good ni and $X_{ni,j}$ is the amount of sector-level bundle $X_{n,j}$ used by ni. These sector-level bundles are formed using the following

CES aggregator of country-specific varieties:

$$X_{n,j} = \left[\sum_{m=1}^{\mathcal{N}} \Omega^X_{n,mj} X_{n,mj}^{\frac{1-\xi_s}{\xi_s}}\right]^{\frac{\xi_s}{1-\xi_s}} \qquad \text{with } \sum_{j=1}^{\mathcal{N}} \Omega^X_{n,mj} = 1,$$

where $X_{n,mj}$ is the amount of output of country-sector mj used by country n, ξ_s is the elasticity of substitution between sector-level varieties, and $\Omega^X_{n,mj}$ is the weight of country-sector mj in the sector bundle for j in country n. Given the production structure of the economy outlined above, it follows that the bilateral flow of intermediate goods produced by country-sector mj and used by country-sector ni is given by:

$$X_{ni,mj} = X_{n,mj} \frac{X_{ni,j}}{X_{n,j}}.$$

2.3 Monetary Policy, Current Account and Exchange Rates

To close the model, we need to assume a monetary policy rule and solve for a world equilibrium. To move forward with this solution, we need to deal with two key variables in an open-economy setting: the current account and the nominal exchange rate.

The majority of the general equilibrium trade literature that analyzyes shocks in a setting similar to ours circumvents the issue of solving for the current account by using "hat algebra" and assuming that current account changes are zero (e.g., see Costinot and Rodríguez-Clare, 2014). However, given that we must solve the model non-linearly, our solution method requires matching observed data (the steady-state values) in a global input-output matrix, which in turn requires matching matching observed initial current account balances and then solving for their post-shock changes. We therefore take a modeling approach similar to the one in Dekle, Eaton, and Kortum (2007, 2008) to solve for model-consistent current accounts.

We further need to solve for each country's exchange rate relative to some numéraire price, since we must convert all domestic factor prices into local currency to check whether the downward wage constraint is violated or not in each sector. Since we solve for a world equilibrium, all input-output accounting is done in terms of a fictitious numéraire common currency. At the steady state, we assume the prices are normalized such that all bilateral exchange rates vis-à-vis the common currency are equal to 1. Upon the realization of the shocks that we feed into the model, these exchanges rates may move so they are no longer at parity with the common currency. The following formalizes how we solve for changes in both current accounts and exchange rates when economies are shocked.

First, we assume all countries are initially at the zero-lower bound in their interest rates ($i = 0$) as observed prior to the Covid shock. With this assumption, the inter-temporal budget formulated in Equation (2) becomes:

$$P_n C_n + P_n^* C_n^* = I_n + I_n^*.$$

Maximizing the inter-temporal utility in Equation (1) yields:

$$I_n = P_n C_n = \underbrace{\frac{1-\beta_n}{\beta_n} I_n^*}_{\text{In local currency}} . \tag{4}$$

Note that this income level is given in the local currency as the inter-temporal budget allocation is made domestically in local currency units given a change in β_n – the aggregate demand shock.

GDP of country n can be calculated by solving for the total value-added (factor income) created in country n. Denoting the rental rate of capital in country-sector ni with R_{ni}, we calculate the GDP of country n in the common currency as:

$$\text{GDP}_n = \sum_i \left(W_{ni} L_{ni} + R_{ni} K_{ni}^* \right). \tag{5}$$

At the world level, global expenditures are equal to global GDP. But for individual countries, because of the trade surpluses and deficits, income may differ from GDP. Calculating GDP and international trade in terms of the common currency, the exchange rate of country n relative to the common currency is denoted by e_n. We express the expenditures in local currency in terms of GDP and trade balance as:

$$I_n / e_n = \text{GDP}_n + \underbrace{\text{Imports}_n - \text{Exports}_n}_{\text{–Current Account}}.$$

To close the model, we assume that the trade balance (current account) is financed by partial ownership of production in foreign countries.[11] To calculate this ownership ratio, we define the

[11] The connection between this approach and that of a current account and savings decisions can be rationalized within a two-period model where countries have access to a foreign bond denominated in the common currency. See

bilateral trade balance between countries m and n as:

$$D_{nm} \equiv \text{Exports}_{m\to n} - \text{Exports}_{n\to m}.$$

A positive trade balance implies that country n is running a deficit vis-à-vis country m. Therefore, country n needs to own a portion of GDP in country m to finance this trade deficit. We define the ownership share of factors/sectors of country n in country m as:

$$\chi_{nm} \equiv \begin{cases} \frac{D_{nm}}{\text{GDP}_m} & \text{if } D_{nm} > 0, \\ 0 & \text{otherwise.} \end{cases}$$

With this definition, we can write the total income of country n in terms of common currency as:

$$I_n/e_n = \text{GDP}_n - \underbrace{\sum_m \chi_{mn}\text{GDP}_n}_{\text{Owned by foreigners in } n} + \underbrace{\sum_m \chi_{nm}\text{GDP}_m}_{\text{Owned abroad by } n}. \tag{6}$$

Equating the spending from the expenditure side (Equation (4)) and the income side (Equation (6)), country n's exchange rate vis-à-vis the common currency is solved for as:

$$e_n \equiv \frac{(1-\beta_n)I_n^*/\beta_n}{(1-\sum_m \chi_{mn})\text{GDP}_n + \sum_m \chi_{nm}\text{GDP}_m}. \tag{7}$$

2.4 Market Clearing

We assume that all good markets clear. Goods can be used as final (consumption) goods and intermediate inputs in many countries. Therefore, we write the goods market clearing condition for country-sector ni as:

$$Y_{ni} = \sum_{m\in\mathcal{N}} \left(C_{m,ni} + X_{m,ni}\right),$$

where country m is the consuming country.

For the factor markets, we take both labor and capital to be sector-specific. Capital is fully

Silva (2023) for details.

utilized and assumed to be at its steady-state level with:

$$K_{ni} = K_{ni}^*.$$

Labor, on the other hand, is subject to shocks. In addition, we assume that there is a downward wage rigidity relative to the steady-state wage. Denoting the amount of available labor for country-sector ni at the time of the shock with $\overline{L}_{ni}$ and given the sector-specific labor assumption implies that:

$$\overline{L}_{ni} \leq L_{ni}^*.$$

Given the downward wage rigidity, there might be slack conditions in a sector's labor market during the shock period. Therefore, the shock-period employment, L_{ni}, maybe be lower than available labor:

$$L_{ni} \leq \overline{L}_{ni}. \tag{8}$$

Finally, the downward wage rigidity necessitates that the wage in a given country sector (W_{ni}) cannot go below its steady-state level (W_{ni}^*) in local currency. The downward wage rigidity condition is then given by:

$$\underbrace{e_n W_{ni} \geq W_{ni}^*}_{\text{In local currency}} \quad \Rightarrow \quad \underbrace{W_{ni} \geq \frac{W_{ni}^*}{e_n}}_{\text{In common currency}} \quad . \tag{9}$$

Optimality implies that at least one of the two preceding inequalities is binding:

$$\left(\overline{L}_{ni} - L_{ni}\right)\left(W_{ni} - \frac{W_{ni}^*}{e_n}\right) = 0. \tag{10}$$

2.5 Solution

To solve the model, we calibrate consumption and input weights, GDP shares and expenditure shares using the OECD Inter-Country Input-Output (ICIO) Tables. We calibrate the CES functions such that the weights coincide with the input and consumption shares. We normalize all prices, wages and rental rates to 1 at the initial steady state. We calculate all changes in common currency units while keeping track of exchange rate movements of countries relative to the common currency, which enables us to convert price changes to local currency when calculating domestic inflation rates.

Figure 1. Structure of enhanced input-output matrix Ω

(a) Ω Matrix

$\Omega =$

	C	Y	Z	VA	X	CB	L	K	Ric	Fut
C	0	0	0	0	0	Ω^C	0	0	0	0
Y	0	0	Ω^Y_Z	Ω^Y_{VA}	0	0	0	0	0	0
Z	0	0	0	0	Ω^Z	0	0	0	0	0
VA	0	0	0	0	0	0	Ω^{VA}_L	Ω^{VA}_K	0	0
X	0	Ω^X	0	0	0	0	0	0	0	0
CB	0	Ω^{CB}	0	0	0	0	0	0	0	0
L	0	0	0	0	0	0	0	0	0	0
K	0	0	0	0	0	0	0	0	0	0
Ric	$1-\beta$	0	0	0	0	0	0	0	0	β
Fut	0	0	0	0	0	0	0	0	0	0

(b) Row / Column Indices

Index	Description	Size	Elasticity
C	Current Consumption	$\mathcal{N}$	1
Y	Goods / Varieties	$\mathcal{N} \times \mathcal{J}$	θ
Z	Intermediate Bundle	$\mathcal{N} \times \mathcal{J}$	ε
VA	Value-Added	$\mathcal{N} \times \mathcal{J}$	γ
X	Country-Sector Bundles	$\mathcal{N} \times \mathcal{J}$	ξ_i
CB	Consumption Bundles	$\mathcal{N} \times \mathcal{J}$	ξ'_i
L	Sector Specific Labor	$\mathcal{N} \times \mathcal{J}$	
C	Sector Specific Capital	$\mathcal{N} \times \mathcal{J}$	
Ric	Ricardian Consumer	$\mathcal{N}$	1
Fut	Future Consumption	$\mathcal{N}$	

Note: All sub-matrix definitions are given in the Sections 2.1 and 2.2. Non-zero sub-matrices are colored and light green colored sub-matrices indicate diagonal matrices.

We provide a unified representation of the model by creating an enhanced input-output table, which is depicted in Figure 1. This generalized input-output matrix integrates households, sector-level outputs, factors and input/consumption bundles that are required for production or used for consumption, with all these entities shown as rows and columns (row and column indices and their sizes are given in Panel 1b). Each row, i, in this matrix corresponds to a single CES aggregator with corresponding elasticity of substitution of σ_i and a price P_i. Given the CES assumption, we can then write the price index for each row as:

$$
\begin{aligned}
P_i^{1-\sigma_i} &= \sum_j \Omega_{ij} P_j^{1-\sigma_i} \qquad \text{if } \sigma_i \neq 1, \\
\log(P_i) &= \sum_j \Omega_{ij} \log(P_j) \quad \text{if } \sigma_i = 1,
\end{aligned}
\tag{11}
$$

where the second equation corresponds to the Cobb-Douglas case.

We write the market-clearing condition for each row entry using information contained in the columns of the Ω matrix presented in Figure 1. For a given column j, we denote its total output by Y_j. This output is used by other entities as inputs or for consumption. X_{ij} is the amount of j used by row i. The market-clearing condition for each row can be written as:

$$P_j Y_j = \sum_i P_j X_{ij} = \sum_i \frac{P_j X_{ij}}{P_i Y_i} P_i Y_i. \tag{12}$$

Using the CES assumption, we then write the optimal input ratio of j in i as a function of relative prices:

$$\frac{P_j X_{ij}}{P_i Y_i} = \left(\frac{P_j}{P_i}\right)^{1-\sigma_i}. \tag{13}$$

Dividing both sides of (12) by global GDP, we express a sector j's output as a function of world output, i.e., its global Domar weight, which is currency free. Hence, we can relate the Domar weights to each other:

$$\frac{P_j Y_j}{\mathrm{GDP}_W} \equiv \lambda_j = \sum_i \Omega_{ij} \left(\frac{P_j}{P_i}\right)^{1-\sigma_i} \frac{P_i Y_i}{\mathrm{GDP}_W} = \sum_i \Omega_{ij} \left(\frac{P_j}{P_i}\right)^{1-\sigma_i} \lambda_i. \tag{14}$$

The Domar weight equations capture the propagation of the consumption of countries, given by Equation (6), down to the payments to factors of production along the global supply chains. Equations (11) and (14) solve for the prices and Domar weights up to a normalization factor, which can then be calculated using the fact that countries return to their steady-state levels in the future with the total world GDP normalized to 1. Since we take the β_n (aggregate demand shock) and I_n^* as exogenous, we also know the expenditure of each country in local currency. Using Equation (7), we can calculate the exchange rates based on the factor Domar weights. Finally, we also respect the downward wage rigidity and labor constraints given in Equations (8), (9) and (10). We use the AMPL/Knitro optimizer to solve for these equations. Since we start by calibrating CES functions with equilibrium prices set to 1, our methodology yields solutions akin to the hat-algebra methodology often used in the trade literature.

2.6 Approximating Inflation

We next generalize Baqaee and Farhi (2022) to the open-economy setting given our model structure. Doing so allows us to provide an analytic first-order approximation of a country's inflation as a function of domestic and foreign shocks. We utilize the Ω matrix in order to capture all necessary information in deriving the approximation to inflation. We briefly sketch out the solution and refer the interested reader to Appendix C for a formal proof.

First, we define the Leontief inverse matrix:

$$\Psi = [I - \Omega]^{-1},$$

where I is the appropriately sized identity matrix. The Leontief inverse captures the direct and indirect dependencies between entities. For country n, its consumption dependencies are captured by the n^{th} row of the Ψ matrix, which corresponds to the households of this country. The entries of this row reflects how much of the output of the output of the corresponding entity accounts for direct and indirect expenditures in country n accounts. These constitute the basis for country-specific Domar weights, which capture the influence of a sector or a factor in the consumption basket of a country. Formally, we define the country-specific Domar weights for each country-sector as:

$$\lambda^n_{mj} = \Psi_{n,mj} \qquad \text{for } mj \in Y.$$

Similarly, for labor, with some abuse of notation we define:[12]

$$\Lambda^n_{mj} = \Psi_{n,mj} \qquad \text{for } mj \in L.$$

We write the corresponding column vector for these Domar weights by dropping the subscripts. With these definitions in hand, we calculate the first-order approximation to the CPI in country n via the following proposition:

Proposition 1. *The first-order approximation to CPI in country n is:*

$$\mathrm{d}\log CPI^n = \underbrace{\mathrm{d}\log I^n}_{AD\ shock} - (\Lambda^n)^T \underbrace{\mathrm{d}\log L}_{Labor\ shock} - (\lambda^n)^T \underbrace{\mathrm{d}\log A}_{Productivity\ shock},$$

where λ^n (Λ^n) is the vector of country-specific Domar weights for country-sector pairs (labor types), A is the vector of sector-specific productivities, L is the vector of labor levels and I^n captures the aggregate demand shock.

[12] Since labor is sector specific, we can index it using the country-sector indices.

Proof. See Appendix C. □

This first-order approximation captures the importance of international linkages since all the terms, namely aggregate demand, labor, and productivity changes, are calculated globally.

The proposition yields several insights. A positive aggregate demand shock to economy n, I^n, creates inflation to the first order. This shock includes both the domestic AD shock and the impact of its exchange rate change vis-à-vis the common currency, which in turn captures changes in other economies and their domestic aggregate demand shocks as well. When considering higher-order solutions, these shocks might induce a slight disinflationary component since aggregate demand shocks may relax the wage-rigidity condition by depreciating the domestic currency. Foreign aggregate shocks might impact inflation through a similar channel. For instance, if there is Keynesian unemployment in a foreign country due to downward wage rigidity, this will create inflation at home. However, if a foreign aggregate demand shock remedies the Keynesian unemployment problem (foreign fiscal stimulus), then inflation decreases at home. Hence, if the US is running a slack labor market because of nominal wage rigidities, a fiscal stimulus in the US might increase inflation in the US but decrease it in the rest of the world.

To examine how the other shocks impact domestic inflation, we first note that households are the terminal nodes of the input-output networks. Starting from the households, we can thus trace back the origin of the goods that are consumed in country n. The Leontief inverse operation captures this path of production to consumption. Applying this Leontief logic, define the share of the output of country-sector mj directly or indirectly (i.e., through supply chains) consumed by households in country n with Y^n_{mj}. Then, the country-specific Domar weight can be written as:

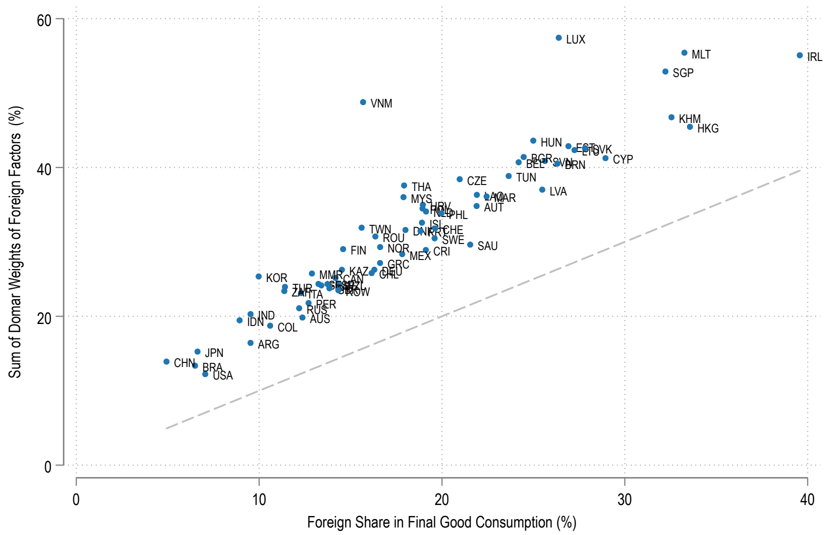
$$\lambda^n_{mj} \equiv \frac{P_{mj} Y^n_{mj}}{I_n}.$$

For each sector, we know the share of the sector-specific labor in its value-added. Then, we can interpret the country-specific Domar weights for labor factors as:

$$\Lambda^n_{mj} = \Omega^Y_{mj,\mathrm{VA}} \Omega^{\mathrm{VA}}_{mj,L} \lambda^n_{mj}.$$

Therefore, Λ^n_{mj}, captures the labor share of the output of mi to directly or indirectly satisfy the

Figure 2. Foreign Share of the Factor Domar Weights



Notes: This figure shows the share of the foreign factors of the Domar weights (Λ^n_{FOR} defined in Equation 15) and foreign share of the final good consumption. Dashed gray line show the 45° line.

consumption in n.

Proposition 1 implies positive productivity changes, $\mathrm{d}\log A$, are deflationary in nature, and shocks to productivity in country-sector mj impact inflation in country n in proportion to λ^n_{mj}. Meanwhile, labor shortages, at home and abroad, are inflationary domestically. The shocks to labor supply in country-sector mj impact domestic inflation in proportion to country-specific Domar weight, Λ^n_{mj}.

Note that labor shortages are inflationary, acting as a cost-push shock regardless of whether they come from (lower labor supply or lower labor demand). Clearly, a negative labor supply shock is inflationary, decreasing the equilibrium level of labor. Suppose, instead, the equilibrium level of labor decreases due to a negative sectoral demand shock. In that case, another sector has a positive labor demand shock since all sectoral demand shocks are relative and factor markets are segmented. Hence, the effect of such labor demand shocks on aggregate inflation is to increase inflation due to factor price inflation in those markets experiencing demand boosts, which is the other side of the coin of employment decreases in the sector experiencing the negative demand shift.[13]

To quantify the potential impact of factor shortages in foreign countries in creating inflation in

[13] Aggregate demand changes act similarly in our model since it is a multi-country, multi-sector model. A shock to all sectors in a particular country can be understood as a sectoral demand shock in a closed economy. Instead of shifting resources away from a sector, it shifts resources away from a country.

country n, we define Λ^n_{FOR}, as the share of foreign factors in satisfying household consumption in country n:

$$\Lambda^n_{\text{FOR}} \equiv \sum_{m \neq n} \sum_j \Lambda^n_{mj} \equiv 1 - \Lambda^n_{\text{DOM}}. \tag{15}$$

The last equality comes from the fact that sum over all factors are equal to 1.

Figure 2 shows the values for Λ^n_{FOR} for all countries present in the OECD's ICIO Tables.[14] The share of foreign factors is higher compared to the direct share of imports in final goods for all countries. Intuitively, this captures the fact that total domestic consumption of foreign goods includes both final goods as well as foreign factors that are "embedded" in all consumption goods (both domestic and foreign) arising from the use of intermediate goods in production. However, these shares vary significantly between countries. For instance, a 1 percent decline in factor levels in foreign countries would potentially result in 0.12 percent increase in the CPI for the US compared to 0.55 percent increase in the CPI of Ireland.

Finally, we also use the Domar weights, λ^n_{mj}, to ask how productivity shocks $\mathrm{d}\log A$ impact domestic inflation (to a first-order) using Proposition 1. According to the proposition, the impact of any country-sector productivity shock will impact aggregate domestic inflation in proportion to that country-sector's Domar weight. This Domar weight captures the direct and indirect use of a given country-sector input into the production of final goods of the ultimate customer, so embeds global supply chain linkages' importance in transmitting foreign sector-level productivity shocks on domestic inflation. This term plays an important role in transmitting the impact of the energy price shocks resulting from the Ukraine-Russian war as we model the change in energy prices as originating from a negative technology shock in Russia's fossil fuels sector. The Domar weight of Russia's energy sector for German consumption is 0.0031, Euro Area consumption as a whole is 0.0025, and for the US consumption is 0.0006. Therefore, the expected effect of an increase in Russian energy price is approximately 5 times higher for Germany and 4 times higher for the Euro Area compared to the US.

[14] See Section 3.1.5 for the description of ICIO Tables

3 Data Description and Construction of Shocks

The model-based quantitative exercises require four sets of shocks: country-level aggregate demand changes, country-sector level factor supply changes, country-sector level demand changes, and global energy price changes. The analyses use data on four "countries": the United States, the Euro Area,[15] Russia, and a Rest of the World composite (ROW) that importantly includes China; and four sectors: durables, non-durables, services, and energy.

We focus on these four countries and sectors given data constraints on sector-level data for ROW countries. Nevertheless, even with this level of aggregation, we are able to capture the key shocks during our analysis period, and how they vary in the cross section. For example our measured shocks are able to capture the stringency of the Chinese lock downs in 2020-2021 (measured by the shocks to the China+ labor supply), the US and European fiscal stimuli of 2021–2022, and the Russia-Ukraine War in early 2022 and its impact on energy prices worldwide. We include Russia and also an "energy" sector to be able to speak to the role of higher energy prices and their implications for the US and, importantly, for the Euro Area, which was the region that, *a priori*, appeared to be one of the country groups that would be the most affected by the energy shock. We map the energy price shock to a decline in Russia's energy sector productivity and study the inflation spillovers of such a shock to other countries. We next describe our data sources and explain details of how we construct the shock series using these data before showing how the shocks evolved over time.

3.1 Data

3.1.1 Aggregate Demand

The model-implied measure of an aggregate demand shock is a deviation in local currency expenditures (I_n) from its steady-state value. We therefore collect cross-country data on nominal expenditures or absorption, depending on data availability, at the quarterly frequency. We then compute growth rates of these series each quarter relative to the base (steady-state) year 2018 to use as shocks in the model.[16]

[15] Note that we rely on Euro Area countries' underlying data and aggregate up.

[16] We choose 2018Q4 as our base period to be able to construct year-on-year model-based inflation rates for 2020. To do so, we require model-predicted price levels for 2019. We could have alternatively used actual data for 2019

United States. We use gross national income (codename: `A023RC1Q027SBEA`) available from the Bureau of Economic Analysis (BEA). These data are available at a quarterly frequency from 2010 to 2022.

Euro Area. Gross national income is only available at a yearly frequency for the Euro Area, which is not appropriate for our empirical application. For this reason, we instead collect data on absorption from EuroStat, which we use as the Euro Area measure of I_n. Aggregate absorption includes household and consumption expenditures, gross fixed capital formation, and imports.

Russia and China+. We construct a measure of the China+'s expenditures by adding up consumption on durables, non-durables, and services from the OECD quarterly national accounts for all countries except the United States and those in the Euro Area. Since data for Russia have not been updated for 2022, we use numbers from the Rest of the World to construct Russia's aggregate demand. While these approximations are not ideal they should not impact the results for the two countries of focus – the US and the Euro Area – given that these countries aggregate demand shocks are more likely to spillover to other countries rather than the other way around given the observed patterns of international trade and relative sizes of stimuli across countries.

3.1.2 Country-Sector Level Factor Supply: Total Hours Worked

The growth rates of total hours, defined as log-deviations from pre-pandemic steady-state values, are used as shocks to potential sector-specific labor *supply*, $\bar{L}_{ni}$. Of course, observed changes in total hours observed in the data are an equilibrium objects and depend on labor demand and labor supply in each sector. Given our modeling assumption of nominal downward wage rigidity, negative changes in equilibrium labor can be rationalized by a decline in labor demand or labor supply. In contrast, positive changes in equilibrium labor can only be rationalized by a combination of labor demand and supply shifts, where a necessary condition is that labor supply shifts at least in the same amount as labor demand. In an extreme case, if labor supply does not shift up while labor demand does, this only creates wage inflation with no effect on the equilibrium level of employment and cannot possibly rationalize increases in total hours worked in equilibrium. As we explain in

instead to calculate the inflation rates, but we wanted to use a consistent methodology throughout the three years of analysis.

detail when discussing the results in the next section, we use the model structure in conjunction with the other set of shocks to disentangle changes in total hours worked into supply and demand. The results support our assumption that changes in observed hours work best capture labor supply shocks.

United States. We use Tables B1 and B2 provided by the Bureau of Labor Statistics (BLS) to collect information to construct our measure of labor supply. These tables contain information on employment and average weekly hours at a monthly frequency, respectively. Since hours in Table B2 are at a higher level of aggregation than those for employment in Table B1, we construct measures of L in the model by multiplying employment in a disaggregated sector by the hours of the aggregate sector. For example, the 'Information sector' contains six sub-sectors in Table B1, but it is only available as an aggregate information sector in Table B2. We thus multiply each sub-sectors employment by the hours of the aggregate sector in Table B2 to obtain a measure of total hours worked in each of the six sub-sectors separately. Our final sample contains information from 2006 to 2022 for 66 sectors that we aggregate up to 4 sectors.

In addition, we also collect information on total private employment (code `CES0500000001`) and hours (code `CES0500000002`) from the BLS and construct total hours worked for the aggregate economy as we did for the sector-level numbers.

Euro Area, Russia and China+. Since sector-level and time series data are not readily available for Russia and China+ for the time span analyzed, we take an indirect approach to construct total hours worked changes for these countries. For the Euro Area, while information is available on hours and employment, they do not necessarily capture the variation we are after due to several pandemic-era laws and regulations that effectively kept measured labor hours and employment from fluctuating very much during this period. For this reason, we use the indirect approach to construct the labor supply shock series for the Euro Area as well.[17]

The indirect approach is as follows. We first regress total hours worked shocks computed at the sector level for the US on the US stringency index from Hale et al. (2021), which aims to capture the strictness of countries' government policies against Covid. Formally, we run the following

[17] Indeed, this indirect approach delivers larger variations in hours worked across sectors than those implied using EuroStat information.

specification for the period 2020m1 to 2022m12:

$$\hat{\varepsilon}(hw)_{st}^{US} = \beta_{0s} + \beta_s S_t^{US} + \nu_{st}^{US},$$

where $\hat{\varepsilon}(hw)_{st}^{US}$ are the total hours worked "shock" in sector s in the United States at time t, constructed as we explained in the previous section, S_t^{US} is the stringency index in the US at time t, and ν_{st}^{US} is an error term. From this regression, we recover the estimated coefficients ($\hat{\beta}_{0s},\ \hat{\beta}^{US}$).

We then project the stringency index of the Euro Area, Russia and China+ using these estimated parameters to get predicted values of total hours worked in each sector for both countries:

$$\hat{\varepsilon}(hw)_{st}^{c} = \hat{\beta}_{0s} + \hat{\beta}_s S_t^{c},$$

where $\hat{\varepsilon}(hw)_{st}^{c}$ is the series total hours worked shocks in country c, sector s at time t and $c = \{\text{Euro Area}, \text{Russia}, \text{China+}\}$.

For Russia we use the direct stringency index provided by Hale et al. (2021). For the Rest of the World, we use a population-weighted average of the stringency index in Hale et al. (2021) by taking this mean across all available countries except the United States and countries belonging to the Euro Area. Importantly, China appears in the stringency index, and as a result, our predictions for the Rest of the World will contain their strict lockdown policies that were a focal point in creating the early supply chain disruptions in 2020. For the Euro Area, we use the population-weighted average of its 19 member countries' stringency indices.

3.1.3 Country-Sector Level Demand: Consumption Expenditure

sector-level demand shocks – changes in $\Omega_{n,j}^{C}$ in the model – are computed as the change in sector-level consumption expenditure shares across non-durable goods, durable goods, services, and the energy sector. Computing the shocks therefore requires cross-country information on disaggregated sector-level consumption patterns at the quarterly frequency.

United States. We use information on personal consumption expenditures from Table 2.3.5U of the Bureau of Economic Analysis version May 2023. This data set contains disaggregated sector-level information on personal consumption expenditures from 1959 to 2022 at a quarterly frequency.

In particular, we use durable, non-durable, services, and energy sector consumption from this table.

Euro Area. We use the information on durables, non-durables, and services from the OECD quarterly national accounts. These data are available from 2010 to 2022 at a quarterly frequency. Unfortunately, the data set does not have information on consumption in the energy sector separately. Since energy consumption is part of non-durable consumption, we assign the change in non-durables to the energy sector.

China+. We use information from the OECD quarterly national accounts to construct sector-level consumption shares for the Rest of the World. We consider all countries except the United States and those belonging to the Euro Area. Consumption series are denominated in local currency for all countries, so to construct a China+ aggregate we convert all series to US dollars using the average exchange rate between 1990 and 2022 per country that we source from the IMF. Finally, we aggregate each consumption series across countries. In doing so, note that we assume the Rest of the World's "fictitious" currency moves with the dollar one-to-one. As was in the case for the Euro Area, we assume energy consumption experienced the same changes as non-durables.

3.1.4 Energy Prices

We proxy energy prices using the energy commodity price index constructed by the IMF (code: `PRNG`). This index contains information on crude oil, natural gas, coal price, and propane price indices and is available at a monthly frequency from 1992 to 2022. We choose this broad index to better capture the potential impact of the Russian-Ukraine War, on countries' inflation rates, and particularly the Euro Area, which heavily depended on Russian natural gas.

3.1.5 Input-Output Matrices, Factor and Consumption Shares

Since we assume two sector-specific factors (capital and labor) in each sector in our quantitative exercise, we need to compute each factor's respective share in nominal GDP. To simulate the model, note that we only need to construct these shares, along with intermediate input expenditure and consumption shares for the initial steady state (the year 2018).

Input-Output Matrices. We use the 2018 inter-country input-output (ICIO) tables from OECD, which contain information for 45 sectors and 66 countries. Given data constraints on other sector-level data (e.g., sector-level hours worked or consumption shares) as well as country coverage for other data series, we aggregate the ICIO tables into our four countries and four sectors of interest. These input-output tables allow us to construct intermediate input linkages at the country-sector level.

Factor Shares. The ICIO tables do not contain information on capital and labor payments at the country-sector level. We therefore supplement the ICIO tables with the structural analysis (STAN) database for the year 2018. This database contains information on labor compensation (labor payments) and gross operating surplus (capital payments). These data allow us to construct the fraction of value added that is paid to labor at the country-sector level for the United States and Euro Area. We aggregate all countries outside of the Euro Area and the United States into a single Rest of the World composite country, sector by sector. Due to data availability, we assume that Russia has the same sector-level labor shares as the Rest of the World. Table A.1 in the appendix shows the numbers we use for each country-sector.

3.2 Aggregate and Sector-level Facts

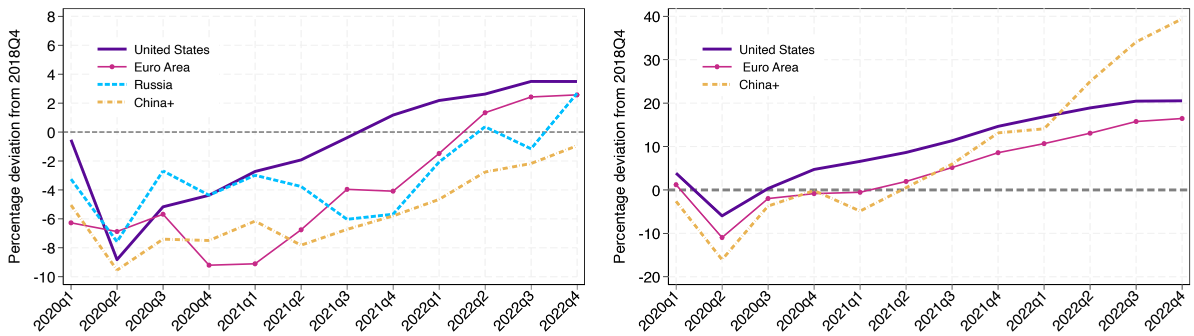
As explained above, we feed in actual data on expenditures, aggregate and sector-level, hours worked, and global energy prices as shocks to our model to recover changes in the sector-level prices and wages, and sector-level expenditure shares together with aggregate prices. It is therefore useful to first examine the time series of the data series used to construct the shock series.

Figure 3 begins by plotting aggregate data, where panel (a) plots the aggregate of log hours worked relative to its 2018Q4 value across countries, and panel (b) plots aggregate demand – log deviation relative to 2018Q4 – across countries.[18] We can see that hours worked declined in all countries to slowly recover their 2018Q4 levels by the end of 2021 for the United States and 2022 for the other countries. Panel (b) of Figure 3 shows the aggregate demand changes

[18] As explained in the earlier section, we construct measures of aggregate demand using nominal expenditures. For the US, we use gross national income. For the Euro Area, we use domestic absorption. For China+, we add up durables, non-durables, and services expenditures from the OECD quarterly national accounts for all countries except the US and those belonging to the Euro Area. Due to data availability, we use the same China+ numbers for Russia.

Figure 3. Aggregate Hours Worked and Expenditures

(a) Total Hours Worked **(b)** Aggregate Demand



Note: These figures plot the log deviations of aggregate time series relative to their 2018Q4 value. Panel (a) of plots total hours worked across countries, while panel (b) plots aggregate demand. See the main text for definitions and data sources of each series. Note that aggregate demand is constructed such that it is equal for Russia and the Rest of the World, so we include only the series for China+.

(nominal expenditures) for the Euro Area, United States, and the Rest of the World. Due to data unavailability of Russia's aggregate expenditure nor nominal GDP, we also use the Rest of the World's aggregate demand shock for Russia. Consistently across countries, aggregate demand plummeted during early 2020 to recover its level in early 2021.

Figure 4 shows the sector-level demand changes as the cumulative growth of nominal expenditures relative to 2018Q4. Several interesting facts jump out. First, services consumption uniformly plummeted across countries at the onset of the pandemic, and barely started to recover in the Euro Area by the end of 2022 and was far below its pre-pandemic level in both the United States and the Rest of the World during the pandemic period. Second, we observe the initial shift in consumption from services to durable goods during early 2020. This shift occurred across all countries, but was by far the largest in the United States. Meanwhile, non-durable consumption growth was relatively larger compared to the growth in durables outside the US.

Figure 5 plots the time series of the energy price shocks. As explained above, the energy index used to construct the shocks contains information on oil as well as natural gas prices. We can see that at the beginning of the pandemic, energy prices were lower than their level in 2018Q4, and began to increase, return to pre-pandemic levels by mid-2021 and then continuing to rise. This pattern is consistent with that described in Gagliardone and Gertler (2023), where oil prices started to rise in mid-2021 and into 2022.

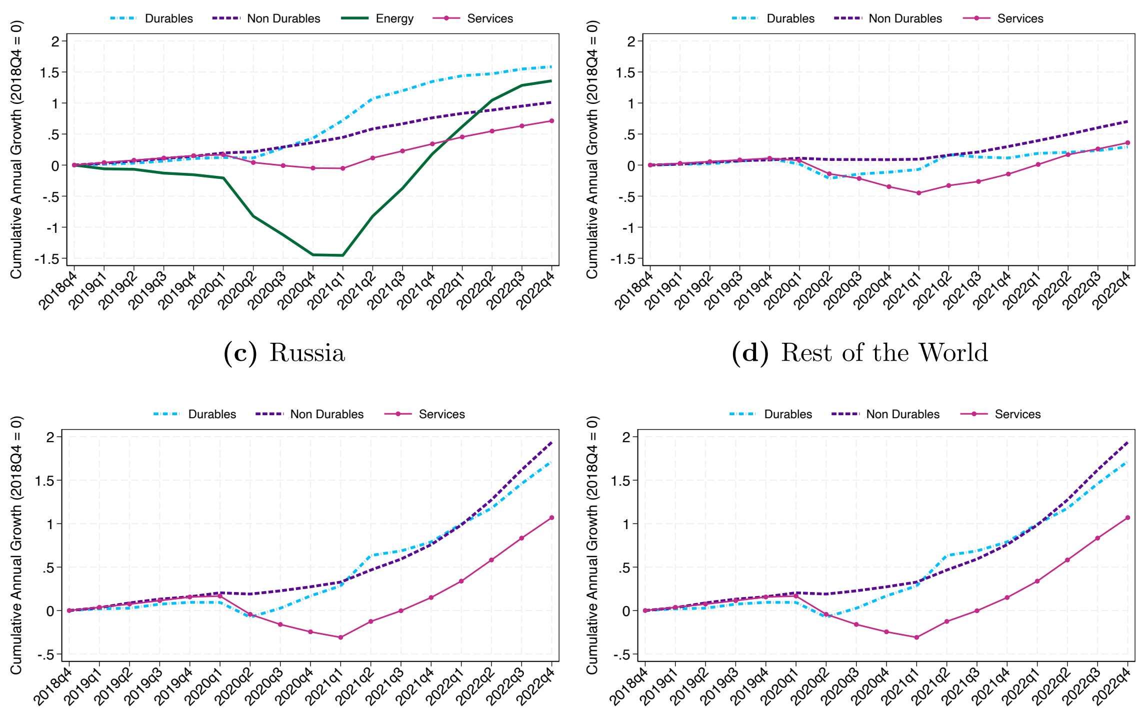
Figure 4. Sector-level Consumption Expenditures

(a) United States

(b) Euro Area

(c) Russia

(d) Rest of the World



Note: This figure plots nominal consumption growth in each quarter vis-a-vis 2018Q4 and cumulated for four different consumption series: durables, non-durables, services, and energy. The cerulean dot-dashed line represents durable consumption. The dashed purple line represents nondurable consumption. The green line represents energy consumption. Finally, the pink dot line represents service consumption. Since we source sector-level consumption for the Euro Area and the Rest of the World from the OECD quarterly national accounts, it only contains information for durables, non-durables, and services. Due to data availability, we use the same behavior of sector-level consumption shares for Russia as that of the Rest of the world. Thus, Panel (c) and (d) are the same.

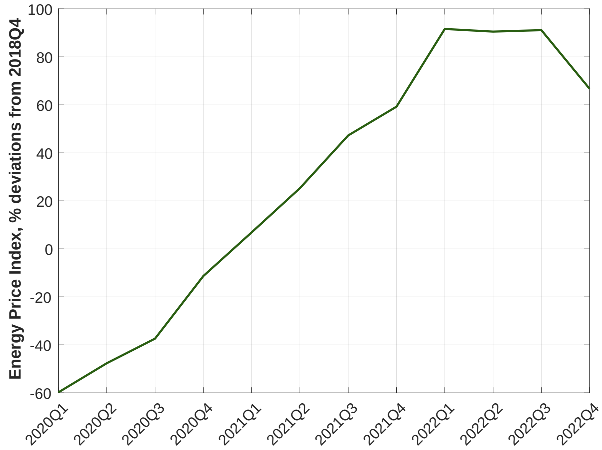
4 Results

This section presents results for model quantification exercises using the data and shock series described above. Given the model's rich consumption and production structures, we must choose several parameters in order to perform our calibrations. Importantly, several of these parameter choices will allow us to control how substitutable factors and goods used for production are with each other, both within and across countries.

Our baseline quantification exercises use the parameter values presented in Table 1. A key assumption that we make in our baseline choice of parameters, based on the recent empirical literature, is that inputs to production have a low degree of substitutability in the short run. We

Figure 5. Energy Prices

(a) Energy



Note: This figure shows the energy price index relative to its value in 2018Q4.

Table 1. Baseline parameter values

Parm.	Value	Source	Related to
θ	0.6	Atalay (2017)	EoS between intermediates and VA
η	0.6	Oberfield and Raval (2021)	EoS across factors
ε	0.2	Boehm et al. (2019)	EoS among intermediate inputs
ξ_s	0.6	Consistent with η, ε	Country-sector level input bundle EoS
ξ_c	0.6	Consistent with η, ε	Country-sector level consumption bundle EoS

Note: 'EoS' stands for elasticity of substitution.

assume complementarities across factors (η) and between factors of production and intermediate inputs (θ). Further, intermediates themselves are difficult to substitute for each other along the whole production process, which is meant to capture the difficulty in substituting between types of inputs (e.g., steel vs. plastic, ε) as well as source of inputs (e.g., Chinese vs. US steel, ξ_s). Similarly, we assume that the elasticity of substitution for sector-level consumption across countries is also low in the short run (ξ_c). We will vary the degrees of substitutability in further exercises to highlight how these elasticities impact the importance of shock transmission to domestic inflation.

Besides elasticities, we also need to take a stand on what kind of shocks we feed into the model each time we estimate it. Table 2 shows the scenarios we consider in this section. In particular, we

Table 2. Shocks and Scenarios

Scenario	Shocks	Countries
Baseline	All	All
Sector-level Supply	Sector-level supply only	All
Sector-level Demand	Sector-level demand only	All
Aggregate Demand	Aggregate demand only	All
Energy	Energy shock only	Russia and China+ Energy Sector
Domestic	All	Domestic country
International	All	Foreign countries
China Supply	Sector-level supply China only	China+

Note: This table shows the different shocks we use in each scenario.

consider eight different model quantification exercises. We explain these as we present their results. Before moving on to discussing the results of the different exercises, we first explain how we use the model structure and quantification exercises to confirm the assumptions we make on the mapping between sector-level supply shocks and the data-shocks we feed into the model.

Sector-level Supply-Only Shock Scenario and Downward Nominal Wage Rigidities. Our model-to-data assumption implies that we consider changes in hours worked at the sector level as if these were shocks to potential sector-level labor supply. In the data, however, changes in sector-level hours worked come from supply and demand forces. We use the model solutions after feeding in shocks to assess if a shock is to labor supply. To help build intuition on this approach, we describe two examples of how the model assesses a labor supply change in a given sector in our quantitative exercises.

Figure 6 presents simple diagrammatic analysis of the forces driving the labor market dynamics in the context of our model over two phases of the pandemic. Panel (a) plots the early phase of the pandemic. The y-axis, W_f, is the wage in the sector, while the x-axis represents the labor quantity, L_f. $\underline{W_f}$ is the lower bound on the nominal wage, $\bar{L}_f$ is the potential labor supply, and L_f^d represents the labor demand. To solve the model, we need to take a stand on the initial equilibrium. Point A represents such equilibrium where labor supply meets labor demand. Starting from point A, an observed fall in hours worked may have been driven by inward shifts in potential sector-level labor supply or labor demand combined with the nominal downward wage rigidity. In the example depicted in the figure, demand shifted by more than supply, thus driving the wage to hit the lower bound at point B. In this case, employment is demand-determined, and there are infinite combinations of potential labor supply shifts, i.e., changes in $\bar{L}_f$, consistent with the economy moving from point A to point B.

Panel (b) shows the late 2021 to 2022 phase, where employment started to recover in some sectors relative to the initial equilibrium. Within our framework and in contrast to the early phase of the pandemic, sector-level employment may only have increased because demand and supply move in tandem. For example, an increase in $\bar{L}_f$ without an accompanying increase in labor demand, L_f^d, puts downward pressure on wages. Since wages cannot fall below $\underline{W}_f$, the rise in $\bar{L}_f$ does not affect wages and employment in equilibrium. Similarly, an increase in labor demand without changes in $\bar{L}_f$ implies that wages must rise without affecting equilibrium employment.

In both cases, we use the model to tell us what the shocks to potential employment, $\bar{L}_f$, are: whenever the full model, where we feed all shocks series, gives a solution where the nominal downward wage rigidity is binding in that sector, we set the potential sector-level supply shock to zero. Otherwise, we assume that hours worked changes in the data maps directly to changes in $\bar{L}_f$. Taking such an approach is conservative: it decreases the role of potential labor supply shocks when hours worked decline in the data relative to 2018Q4. Thus, our numbers on sector-level supply changes have to be considered as a lower bound on the role of sector-level supply on inflation under this approach.

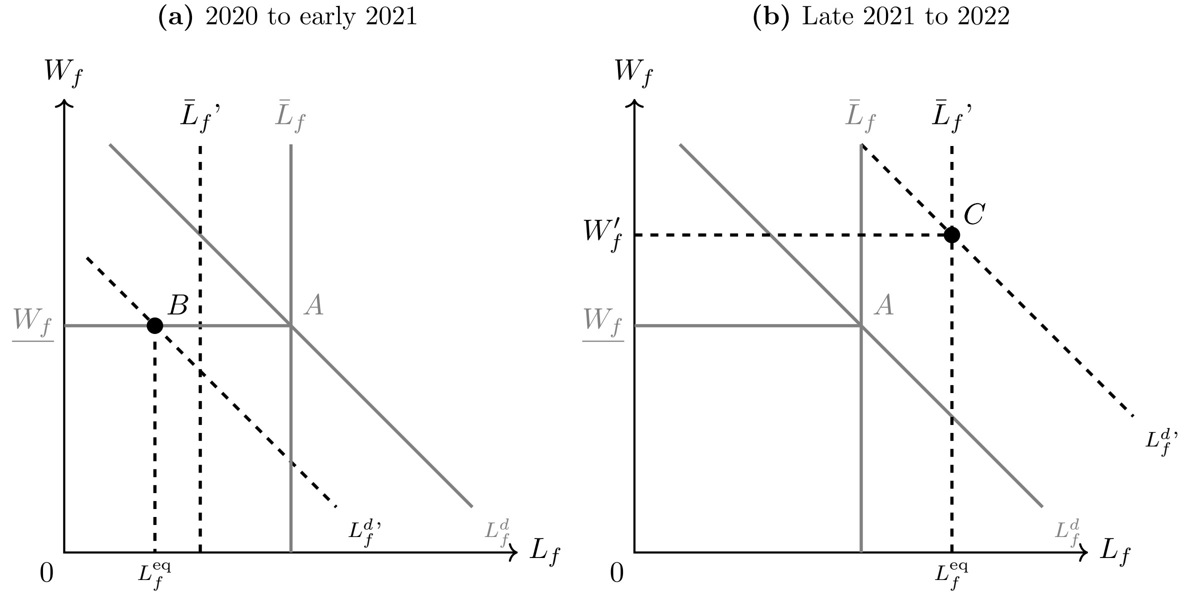
4.1 Baseline Quantification Exercise

Figure 7 begins by plotting quarterly CPI inflation rates for the model calibration, using all shocks to all countries, and data for the United States and the Euro Area, in panels (a) and (b) respectively. Both sets of inflation rates are calculated as year-on-year annual growth rates.[19]

We plot actual inflation with the solid black line while the blue diamonds are the model-generated inflation rates that are calculated by feeding in the shock series quarter-by-quarter. We further highlight two periods with pink diamonds: (i) the Covid Lockdown, and (ii) the Rebound resulting from economies reopening. The magnitude of shocks during these periods, particularly

[19]The model gives the price level in deviation from steady-state. We convert them to year-on-year annual growth by taking the annual (log) difference between the model-predicted post-shock price levels. The resulting series is our model-based inflation.

Figure 6. Sector-level Labor Markets under Nominal Downward Rigidity

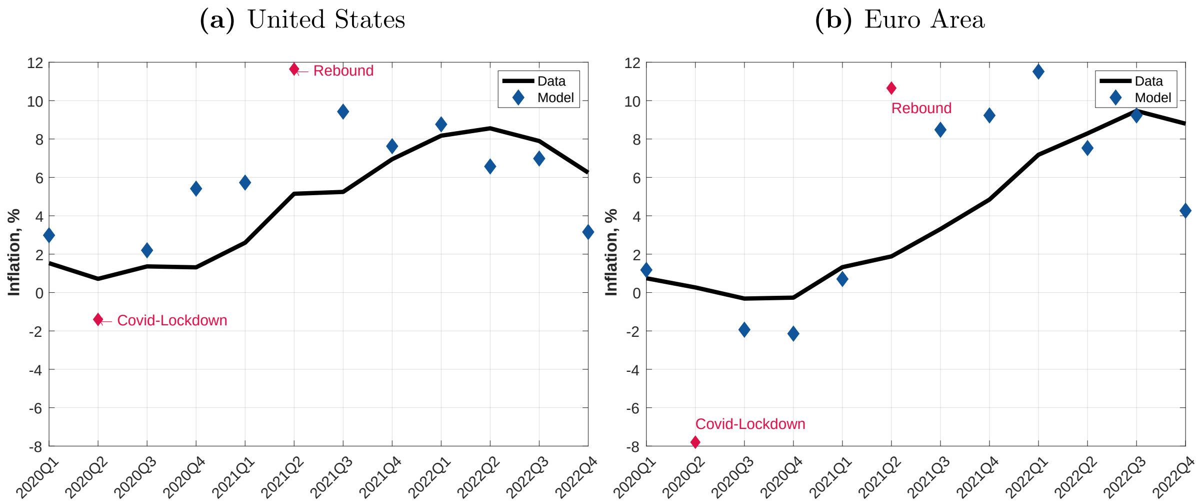


aggregate demand, are an order of magnitude larger than economic shocks witnessed in recent memory (e.g., compared to the Global Financial Crisis), and we therefore put less weight in the model being able to match observed inflation during these periods.[20] The model still performs remarkably well in matching observed inflation over the 2020q1–2022q4 period for both the US and the Euro Area.

Figure 8 next shows that our model calibration produces USD/EUR exchange rate dynamics similar to those observed in the data throughout the sample period. Of note, the model matches the USD depreciation in 2020 and then its appreciation since early 2021. These exchange rate dynamics correspond to the movements in the US current account, which the model also has success in matching. Figure 9 plots the model and data current-account-to-GDP ratio over time for the US. As can be seen, the current account deficit widened in 2020, improved in 2021, and then widened again in late 2021–2022. This pattern matches well with the pattern of movements in US aggregate demand and savings, which originally increased during the lockdown but then started to fall given aggregate demand stimulus (See Aggarwal et al. (2023), Gourinchas et al. (2021) for similar current account dynamics).

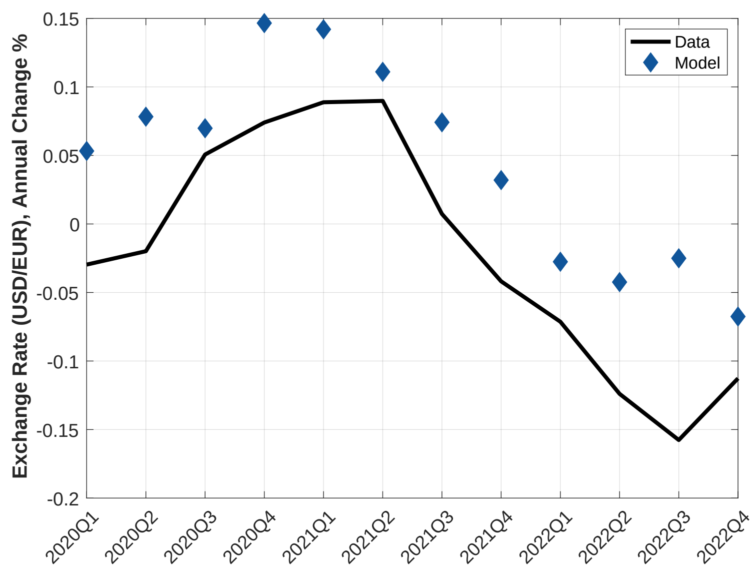
[20] It is also debatable how well national statistical agencies were able to measure economic series, such as GDP or aggregate expenditures, during the Covid lockdown.

Figure 7. United States and Euro Area Inflation Rates: Baseline Model vs. Data



Note: This figure shows annual inflation implied by the model (blue diamonds) relative to the headline CPI inflation in the data (black solid line) when feeding the model with all shocks. The pink diamond in 2020Q2 highlights the Covid lockdown period, while the pink diamond in 2021Q2 highlights a "base effect" that exist in macroeconomic time series, as economic activity had a huge rebound relative to the Covid lockdown phase once the economy reopened.

Figure 8. USD-Euro Exchange Rate: Model vs. Data



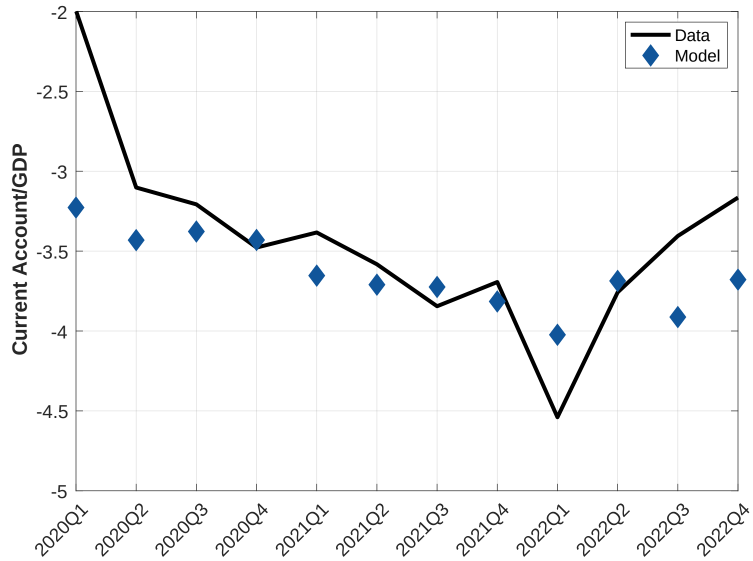
Note: This figure plots the annual percentage change in the USD/EUR exchange rate implied by the baseline model (blue diamonds) and the observed change in the data (black solid line) for comparison.

4.2 Shock Decomposition

We next provide a decomposition of the "all shock" inflation numbers that were generated by the model shown in Figure 7. To do so, we re-estimate the model by applying each shock one-by-one

Figure 9. Current Account-to-GDP Ratio: Model vs. Data

(a) United States



Note: This figure shows the current account-to-GDP ratio implied by the baseline model (blue solid line). We also plot the current account-to-GDP ratio as in the data (black solid line) for comparison.

(for all countries). Figure 10 shows the output for these exercises for the US and Euro Area in panels (a) and (b), respectively. Before describing the full sets of results, it is worth noting that the sum of predicted inflation rates of the different "shock experiments" need not equal the inflation rate of the "all shock" model results reported above, since the solution to that model captures non-linear interactions generated by applying all shocks simultaneously.

We begin by considering the impact of sector-level supply shocks (the blue dots) in isolation. Two main patterns emerge, both for the US and Euro Area. First, sector-level supply shocks were inflationary early in the pandemic. Thus, in the absence of these shocks, there would have been more disinflation early on than observed in the data. Ssecond, we see that without the expansion of sector-level supply in early 2022 as supply chain bottlenecks began to clear up and workers began returning more to the labor market, inflation would have been even higher in both the US and the Euro Area.

Second, we examine the role of sector-level demand shocks (the yellow crosses). The shocks capture the consumption switching across sectors that took places as economies closed and then reopened. Interestingly, the substitution to goods consumption in the US early in the pandemic had an inflationary effect, but the rebalancing later as the economy reopened did not have a disinflationary effect. Meanwhile, the demand substitution does not appear to have played a role in driving the inflation in the Euro Area throughout the sample period.

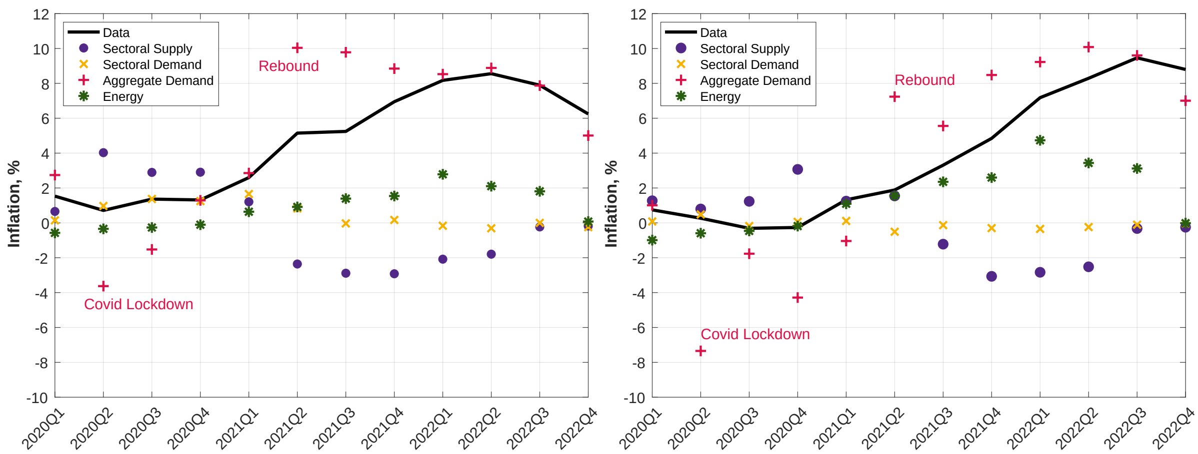
Third, we explore the role of aggregate demand shocks (pink plus signs) in the evolution of inflation across countries. According to our quantification exercises, these shocks clearly played an important role in driving inflation over the period. Notably, the model captures the impact of the fall in aggregate demand and its disinflationary forces early on in the US as is also found in Baqaee and Farhi (2022). Interestingly, these negative aggregate demand shock forces appear to have played an even greater role in the Euro Area early on (irrespective of including in the 'Covid Lockdown' point). The reopening rebound and various expansionary policies then had a large inflationary impact in both countries. The model results show that the positive aggregate demand shock has a larger impact in the US vs. the Euro Area, which matches up well with the narrative of the differential impact of stimulative policies in the two regions over this period. Looking at the end of the sample period, a fall in the aggregate demand shock explains the fall in inflation at the end of 2022.

Finally, the green stars denote the model-predicted inflation arising from energy shocks. These shocks play a minor role early on in the pandemic and are if anything disinflationary. Moving into 2021 and the Russia-Ukraine War, we see that energy shocks start to exert an upward pressure on prices. Looking at the period where this effect was at its peak, 2022Q1, we find that the model predicted energy inflation is almost twice as large for the Euro Area and than in the US; 5.5 vs. 3 percent.

International Spillovers. We next investigate the role of international spillovers, under our baseline flexible exchange rate regime, on domestic inflation in Figure 11. Comparing the domestic (orange crosses) and international (purple pluses) points US and the Euro Area, we see that model-based inflation is mostly a domestic shock-driven phenomenon. This result defers from our earlier work, where the model was solved under fixed exchange rates, and we found that external shocks played a larger role for the Euro Area: we found that two-thirds of inflation in Europe was explained by external shocks (di Giovanni, Kalemli-Özcan, Silva, and Yıldırım, 2022). International shocks have less of an effect in our current model as exchange rates are allowed to change endogenously.

Figure 10. United States and Euro Area Sources of Inflation: Shock Decomposition

(a) United States **(b)** Euro Area

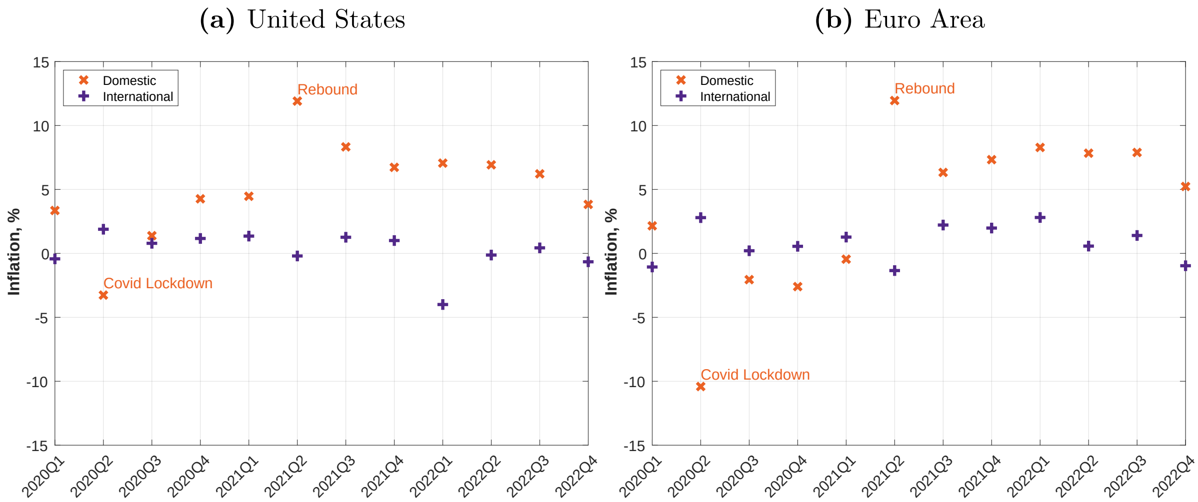


Note: This figure shows annual inflation in the data (black line) relative to the data when feeding the model with all shocks and counterfactual scenarios where we feed in one type of shock at a time. The pink + in 2020Q2 highlights the Covid lockdown period, while the pink + in 2021Q2 highlights a "base effect" that exist in macroeconomic time series, as economic activity had a huge rebound relative to the Covid lockdown phase once the economy reopened.

Of note, the impact of aggregate demand shocks on inflation are neutered by the corresponding exchange rate movements. However, the effect of international shocks are still more important for Euro Area countries compared to the US, which is consistent with the differences in the foreign-factor component of consumption depicted Figure 2 (as well as our earlier findings under a fixed exchange rate modeling assumption). In other words, the foreign factor content of European output is larger than that of the US, so the Euro Area inflation is more impacted by shocks to foreign factors (i.e., foreign supply shocks that are transmitted by the global production network). This result is notable is 2022Q1, which picks up European's reliance of imported petroleum products in production.

The Role of Chinese Lockdown Shocks. Chinese lockdown policies played an important role in generating supply chain bottlenecks throughout the world, helping to generate supply-demand imbalances that arguably helped to fuel inflation across countries. We examine the impact of the "Chinese supply shocks" using the model by feeding in China-specific labor supply shocks in the model, while omitting other shocks. To be precise, we apply the China sector-level labor supply shocks to all countries in the China+ aggregate and simulate the model. Figure 12 presents results of this quantification exercise of the US and the Euro Area in panels (a) and (b), respectively. This

Figure 11. United States and Euro Area Sources of Inflation: Domestic and International Shocks



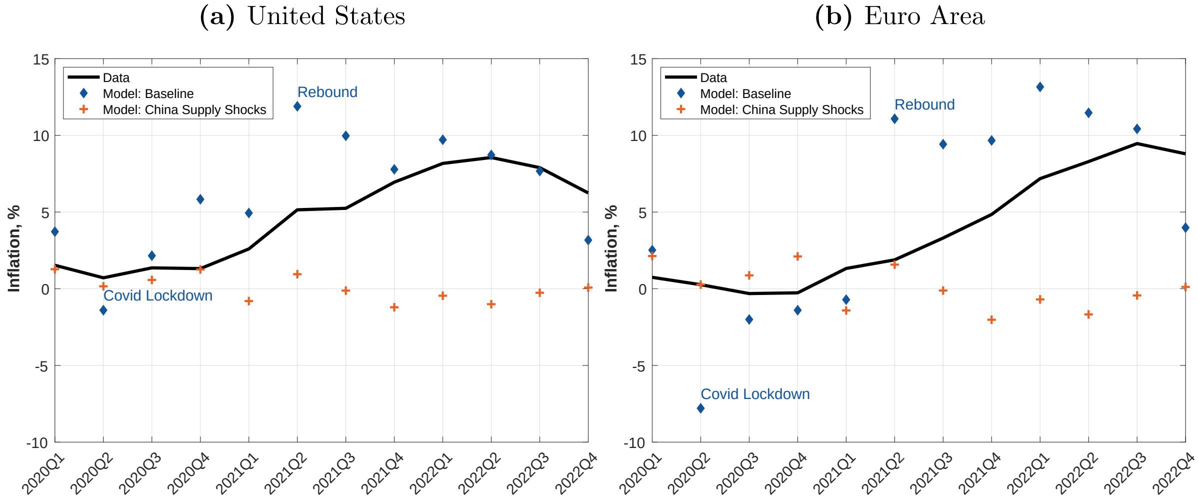
Note: This figure shows annual inflation when feeding only domestically originated shocks (orange x) relative to international shocks only (purple $+$). 2020Q2 highlights the Covid lockdown period, while 2021Q2 highlights a "base effect" that exist in macroeconomic time series, as economic activity had a huge rebound relative to the Covid lockdown phase once the economy reopened.

exercise shows several interesting patterns. First, the China shock had a non-negligible impact in both regions early in the pandemic, with its impact dampening out by the end of 2021. Second, we see that the impact of the China shock was fairly large during the re-opening rebound period, 2021Q4, which highlights the peak supply-demand imbalances as economies began to reopen. Finally, contribution of the China supply shock to Euro Area inflation was as large, if not larger, than its contribution to the United States. This captures the fact that while the US increased demand for goods relatively more than the Euro Area over the pandemic, the Euro Area's foreign content of consumption (see Figure 2) is larger than that of the US, so it was more impacted by negative supply shocks to factors in China. This result corresponds to the overall results we found on international spillovers in Figure 11.

4.3 Production and Trade Elasticities

We next investigate the quantitative importance of varying elasticities of substitution within the global network on the amplification of shocks on domestic inflation. We first examine how changing the elasticities of production impact the amplification of shocks to domestic inflation, and then move on to conducting a similar exercise for trade elasticities.

Figure 12. United States and Euro Area Inflation Rates: The Role of China



Note: This figure shows annual inflation implied by the baseline model (blue diamonds) relative to the headline CPI inflation in the data (black solid line) when feeding the model with all shocks and when feeding all shocks but sector-level supply (orange plus). 2020Q2 highlights the Covid lockdown period, while 2021Q2 highlights a "base effect" that exist in macroeconomic time series, as economic activity had a huge rebound relative to the Covid lockdown phase once the economy reopened.

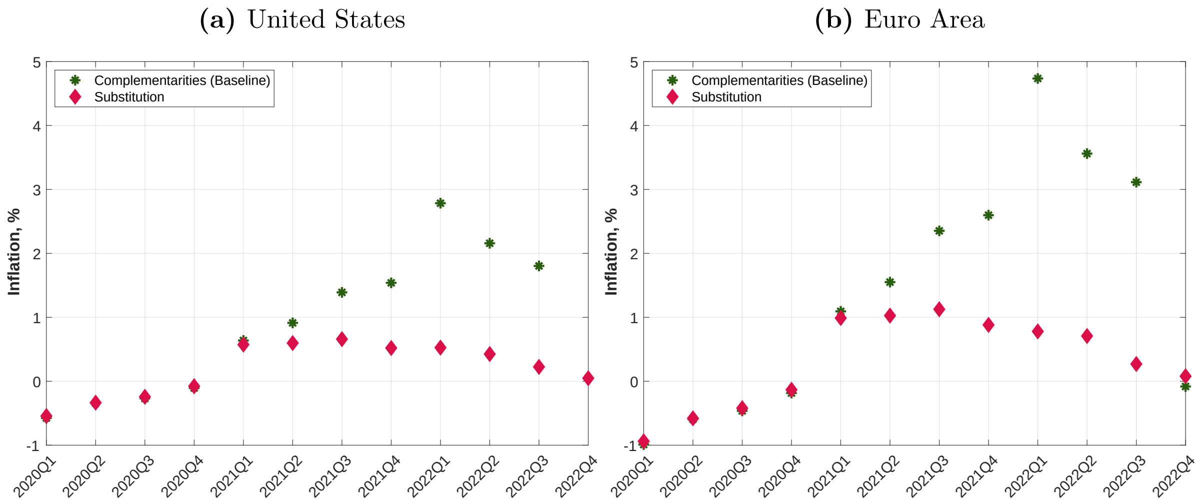
We focus on how varying the elasticities of substitution in production impacts the effects of shocks to the energy sector. To do so, we vary the elasticity of substitution and reallocation of factors across sectors commonly used in the literature. The new higher-substitutability elasticities we feed into the model are $(\eta, \varepsilon, \theta) = (1.5, 1.5, 1.5)$.[21]

Figure 13 presents simulation results when we vary production elasticities. We plot our baseline ('Complementarities') calibration with the one that imposes a higher elasticity of substitution across factors used in production ('Substitution') – note that these substitution parameters impact allocation across country-sector pairs. As can be seen, allowing producers to more easily substitute across factors substantially dampens the impact of energy shocks on inflation in both the US and the Euro Area. The impact of this difference in elasticities on inflation is notably larger in the Euro Area during the Russia-Ukraine War, which is unsurprising given the Euro Area's higher exposure to the energy price shock.

We next analyze the role of trade elasticities. To do so, we vary the trade elasticity parameters

[21]We choose 1.5 for these elasticities because it represents the upper end of available short-run estimates in the literature. $\theta = 1.2$ in the baseline of Carvalho, Nirei, Saito, and Tahbaz-Salehi (2021) and $\eta = 1.5$ in Table 1 specification (vi) of Karabarbounis and Neiman (2014). Estimates of ε are typically below one at any horizon. For example, Peter and Ruane (2023) find this elasticity to be 0.6 at a 7-year horizon, and Boehm, Flaaen, and Pandalai-Nayar (2019) estimate this to be 0.03 in the short run. We take the extreme symmetric assumption $(\eta, \varepsilon, \theta) = (1.5, 1.5, 1.5)$ for parsimony to highlight the importance of complementarities in production.

Figure 13. Production Complementarities and Energy Shocks Transmission to Inflation

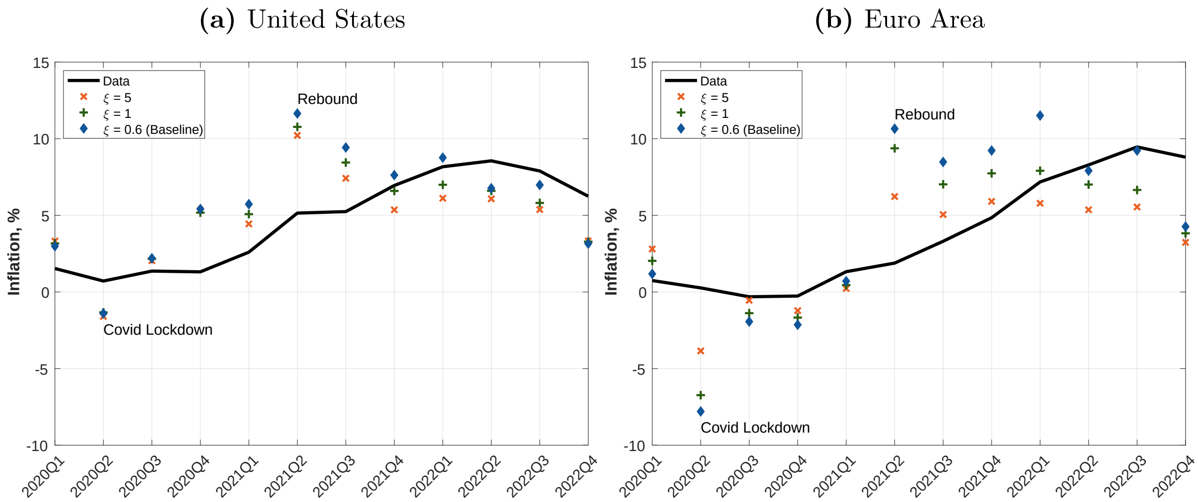


Note: This figure shows annual inflation implied by the baseline model (green $*$) relative to the model with high elasticities of substitution (pink diamonds).

ξ over three possible values, $\{0.6, 1, 5\}$, for both the importing of consumption and intermediate goods. Varying these elasticities, conditional on holding production elasticities at their baseline values, is meant to capture how a change in the ease of access to different markets impacts the transmission of shocks to domestic inflation. These experiments help to further gauge the impact of global supply chain bottlenecks on inflation. Specifically, varying the elasticities from low (short-run) to high (long-run) values will capture the ability of countries to substitute between suppliers for goods and services.

Figure 14 presents the resulting inflation patterns from varying trade elasticities in the US and Euro Area. Unsurprisingly, allowing for greater substitution of consumption and intermediate goods from source countries along the global supply chain dampens the impact of shocks on domestic inflation. The impact of increasing elasticities is quite small for the US but has a substantial impact on the model exercises for the Euro Area. The higher elasticity dampening effect on the propagation of shocks to Euro Area inflation captures Europe's higher exposure to foreign factors of production that is embedded in domestic consumption, as measured in the foreign Domar weights in Figure 2.

Figure 14. United States and Euro Area Inflation Rates: The Role of Trade Elasticities



Note: This figure shows inflation numbers when we introduce all shocks under different trade elasticities $\xi_c = \xi_s = \xi$. Blue diamonds represent our baseline model. Orange x use a high trade elasticity $\xi = 5$. Green + assumes a unitary trade elasticity, $\xi = 1$.

4.4 Real Wages

This subsection examines how closely the model is able to match real wages movements, both in the aggregate and across sectors. We focus on the US since detailed wage data at the sector-level level is more readily available for this country. Specifically, we use the non-farm business sector hourly compensation to measure nominal wages. These data come from FRED (code `COMPNFB`). We deflate the series using headline CPI to obtain our measure of real wages from the data. We similarly compute the real wage from the model by deflating the aggregate nominal wage by the overall price index.

Figure 15 compares the behavior of real wages in the data and those generated by baseline model quantification exercise by plotting the year-on-year growth rate of real wages. The black line represents the data, and the blue diamonds depict model predictions. The model-generated series tracks the evolution of real wages observed in the data quite closely over the analyzed period for the United States, and it is consistent with the large increase in the real wage during 2020 and its subsequent decline over 2021–2022.

We next take advantage of the model structure to examine its performance at the disaggregated level. This would not be possible without modeling the supply side across several sectors of the economy (along with the input-output structure) – an advantage of our methodological approach absent in much of the other literature focusing on the pandemic inflation period. Specifically, we are able to study how well the model matches the evolution of sector-specific real wages. Performing this exercise is important because when labor is sector-specific and immobile across sectors, a key relative price is the real wage in units of the sector-specific price. For ease of exposition, we aggregate sectors into goods (durables, non-durables, and energy) and services.[22] Figure 16 compares the model to the data for sector-level real wages in the goods sector in panel (a) and the service sector in panel (b).[23]

Overall, in the model and the data, goods and services real wage growth was positive during 2020 before declining over the 2021–2022 period. The model matches the overall behavior of real wages in the goods sector, as shown in panel (a), with an initial positive growth and subsequent fall in the real wage. While we over-predict real wage growth from 2020 until mid-2021, the model captures the magnitude of the decline in real wage growth from mid-2021 onward. In contrast, panel (b) shows that the model results closely track the real wage growth in the service sector in 2020 but underpredict this sector's real wage growth starting in 2021. This result suggests that the service sector price increases faster than its wage. In our model, this means that other factor prices — wages and, notably, the price of capital in other sectors — went up more than wages in the service sector.

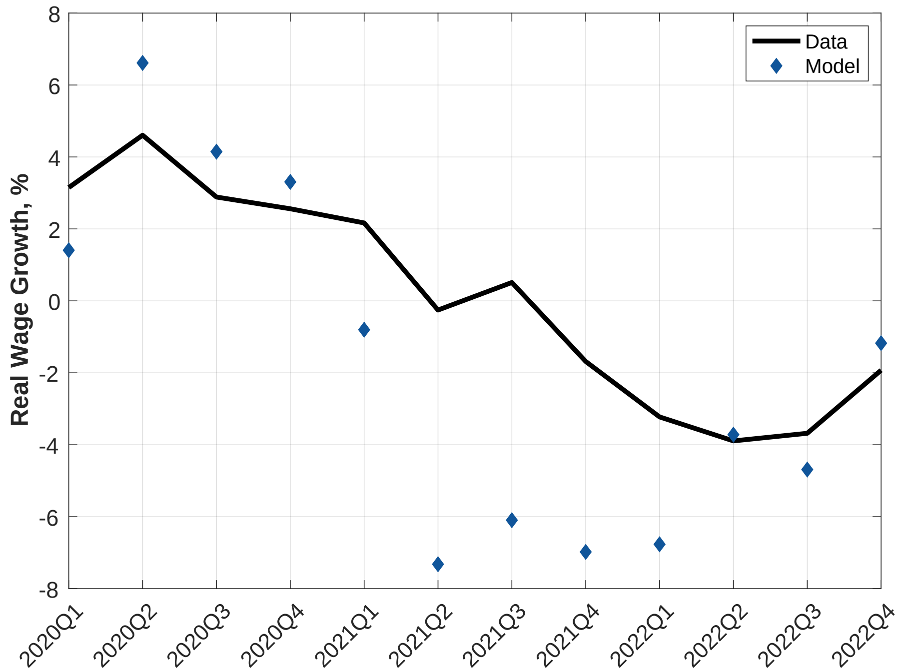
Figure B.2 in the appendix plots the nominal wages for the goods and services sector. The model predicted behavior of nominal wages in the service sector (reported in panel (b)) closely tracks the time series of nominal wages observed in the data. Thus, the reason why the model does not track real wages for services is due to the model overstating increases in the service sector's price growth, especially during 2021. As we analyze in the following subsection, the services sector price indeed increased more in our model than in the data. We believe this is due to missing granularity in sector-level prices in the data as the latter combines the increase in, for example, grocery delivery prices with the decrease in restaurant prices, leading to a flat pattern of the service sector price.

To sum up, the sector-specific real wage growth patterns are consistent with sector-level labor supply declines that initially put upward pressure on sector-level real wages during the initial phase of the pandemic, coupled with increases in labor demand driven by positive aggregate demand

[22] We do this for the model and data based on each sector's nominal wage and price levels.

[23] Figure B.1 shows each sector nominal wages deflated by the overall price level instead of sector-specific prices. We find similar results.

Figure 15. Real Wage Changes



Note: This figure shows year-on-year real wage growth rate. Nominal wage corresponds to total private sector hourly earnings series from the Bureau of Labor Statistics, codes `CES0500000003`. We deflate this measure using headline CPI. The black line represents the data. Blue diamonds are model-based predictions.

shocks starting at the end of 2020. The story in 2021 and 2022 is a recovery in sector-level employment supply and demand, with a corresponding decline in real wages. These findings suggest that sector-level labor supply played a key role in real wage dynamics during 2020–2022.

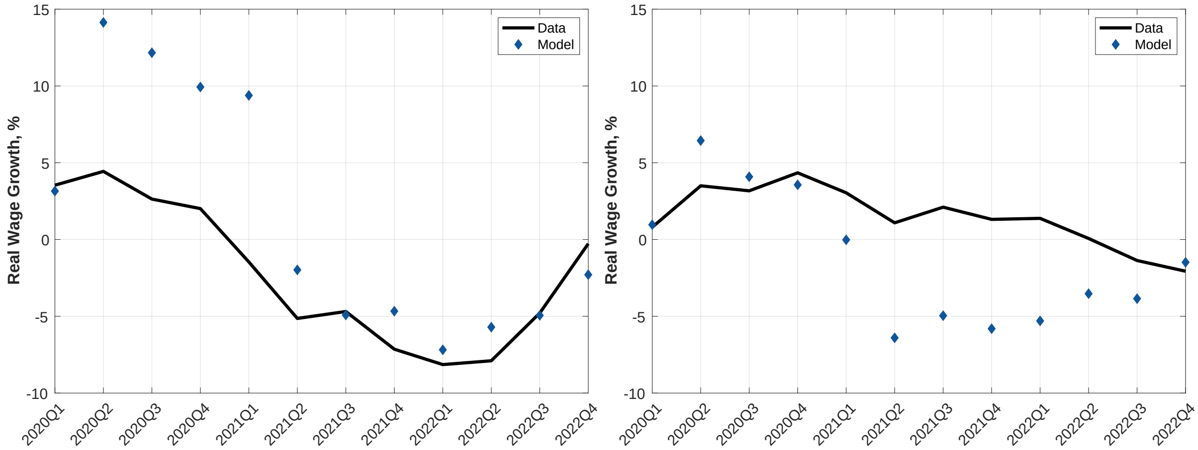
4.5 Sector-level Price Inflation

We next examine how well the model matches observed sector-level price movements, focusing on the US again. Figure 17 plots US sector-level price inflation. Overall, the model-predicted inflation rates match the data well for the US. However, as in the case of aggregate CPI inflation presented above, the model tends to over-predict inflation quite a lot during the reopening period.[24] Goods inflation (durables, non-durables, and energy) was initially weakly positive and increased to stabilize around 2021 in both the non-durables and energy sectors, while it declined in the durables sector during this period. The continuous increase in the non-durables and energy sector prices helps to explain the real wage growth decline in the goods sector we observed since 2021 in panel

[24] See Figure B.3 for the shock decomposition of the sector-level price inflation series.

Figure 16. Sector-Specific Real Wages Growth

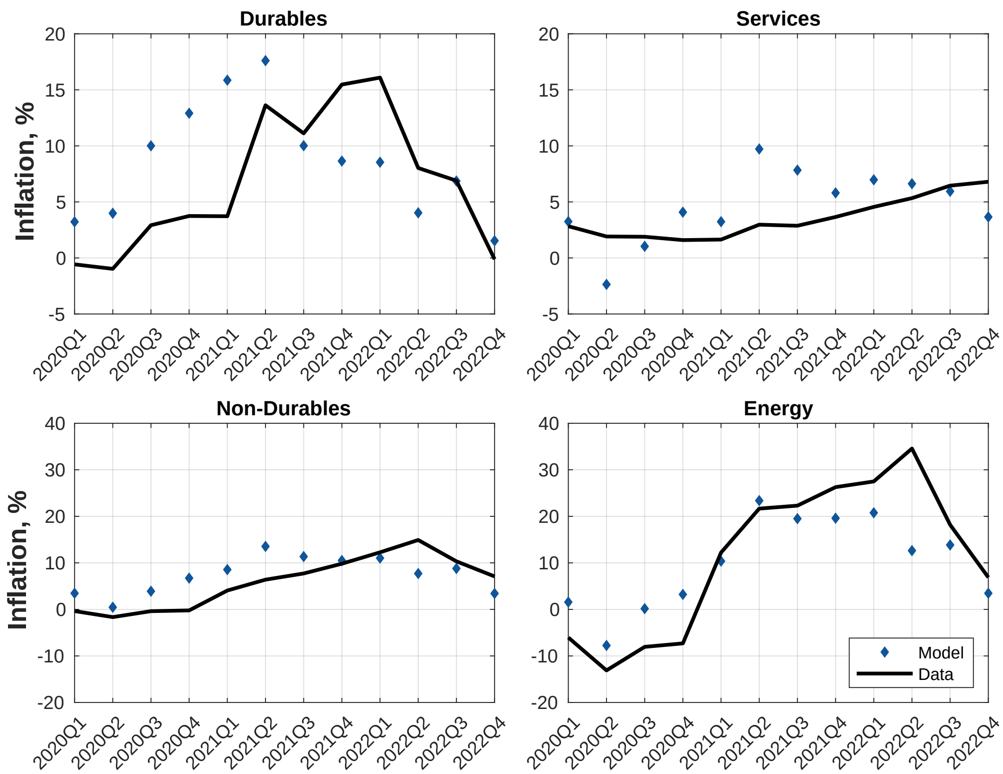
(a) Goods **(b)** Services



Note: This figure shows sector-specific real wage growth. Black lines represent the data, while blue diamond represents the model. We compute real wages by deflating nominal wages in that sector by the sector-level price. All numbers are year-on-year growth; each panel shows the real wage growth for a different sector. In the model, we aggregate nominal wage growth across the durables, non-durables, and energy sectors to construct nominal wage growth in the goods sector. We subtract goods inflation from nominal wage growth to construct real wage growth in the goods sector. We source sector-level nominal wages from the Bureau of Labor Statistics average hourly earnings series with codes: `CES0600000003` (goods) and `CES0800000003` (services). We source sector-level prices from FRED with codes: `CUSR0000SAC` (goods) and `CUSR0000SASLE` (services).

(a) of Figure 16. Conversely, the model predicts a significant and persistent increase in service inflation, which is larger than in the data from 2021 onwards. As we have articulated in the above section, we believe this is due to the missing granularity in sector prices data where higher prices of shelter and online services and lower prices in contact-intensive services sectors are averaged out in service sector prices. These dynamics in service prices help explain why the model-predicted real wage growth in the services sector reported in panel (b) of Figure 16 was negative during the latter half of the sample period.

Figure 17. United States Sector-level Price Inflation



Note: These figures plot year-on-year sector-level model-based inflation rates implied for all shocks (blue diamonds) and actual inflation (black lines) for the US. Data comes from FRED. Codes: Durables `CUSR0000SAD`, Non-durables `CUSR0000SAN`, Services `CUSR0000SASLE`, Energy `CPIENGSL`.

5 Conclusion

This paper estimates a multi-country multi-sector general equilibrium model to quantify the drivers of global and domestic inflation during 2020–2023. Given the global nature of the model, we also measure inflation spillovers across countries. The baseline quantification exercise produces aggregate inflation rates that match those observed in the data across countries, as well as being able to explain movements in sector-level prices and wages that are similar to those observed in the data. The model is also able to match endogenous exchange rates and current accounts in the data. This is an important feature given the open economy dimension of our work that takes into account both trade and production linkages globally.

The model further allows us to conduct a shock decomposition exercise, which quantifies the drivers of inflation in each phase of the pandemic-inflation period. This exercise shows that inflation began due to pandemic-related supply shocks in factor markets and increased further due to expansionary fiscal and monetary policies that stimulated aggregate demand. The reallocation of consumption across sectors combined with energy shocks also played important roles in the global amplification of shocks together with the complementarities in production. We highlight this narrative that we quantify in the introduction by a quote from Chairman Powell. The overarching policy implication of our paper is that, in a world with more supply shocks (a fragmented and de-globalized world), at sector or at the aggregate level, we will see more inflation, regardless of the fact that monetary policy stays restrictive.

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A Additional Tables

Table A.1. Sector-level Labor Shares

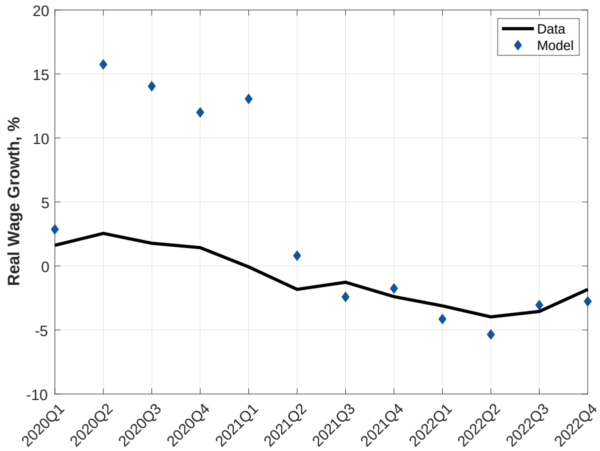
	Euro Area	United States	Rest of the World	Russia
Durables	0.61	0.57	0.43	0.44
Non-Durables	0.57	0.44	0.56	0.56
Services	0.54	0.59	0.68	0.68
Energy	0.39	0.11	0.25	0.25

Note: This table shows the share of value-added that accrued to labor. Value added is compensation to employees (labor) plus gross operating surplus (capital). Data comes from the Structural Analysis Database (STAN) year 2018 to be consistent with the ICIO tables we use to compute other relevant shares.

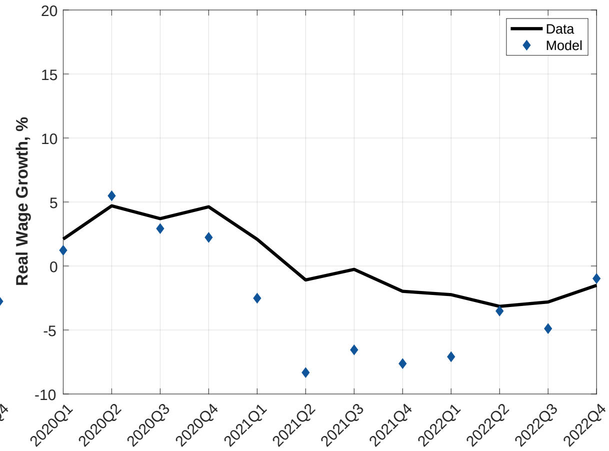
B Additional Figures

Figure B.1. Sector-Specific Real Wages Growth: Sectoral wages deflated by aggregate price

(a) Goods



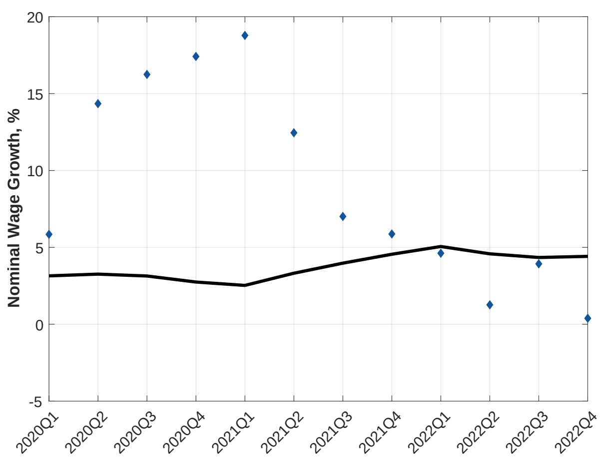
(b) Services



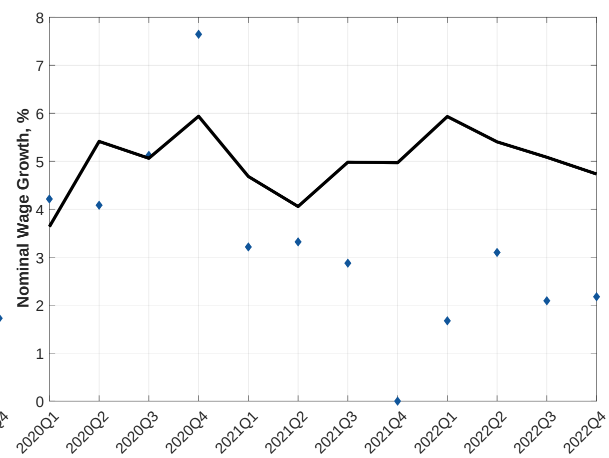
Note: This figure shows sector-specific real wage growth. Black lines represent the data, while blue diamond represents the model. We compute real wages by deflating nominal wages in that sector by the aggregate price. All numbers are year-on-year growth, and each panel shows the real wage for a different sector. In the model, we aggregate nominal wage growth across the durables, non-durables, and energy sectors to construct nominal wage growth in the goods sector. We subtract overall inflation from this number to construct real wage growth. We source sector-level nominal wages from the Bureau of Labor Statistics average hourly earnings series with codes: `CES0600000003` (goods) and `CES0800000003` (services). Aggregate price corresponds to code `CPIAUCSL` (headline CPI).

Figure B.2. Sector-level Nominal Wage Growth United States

(a) Goods

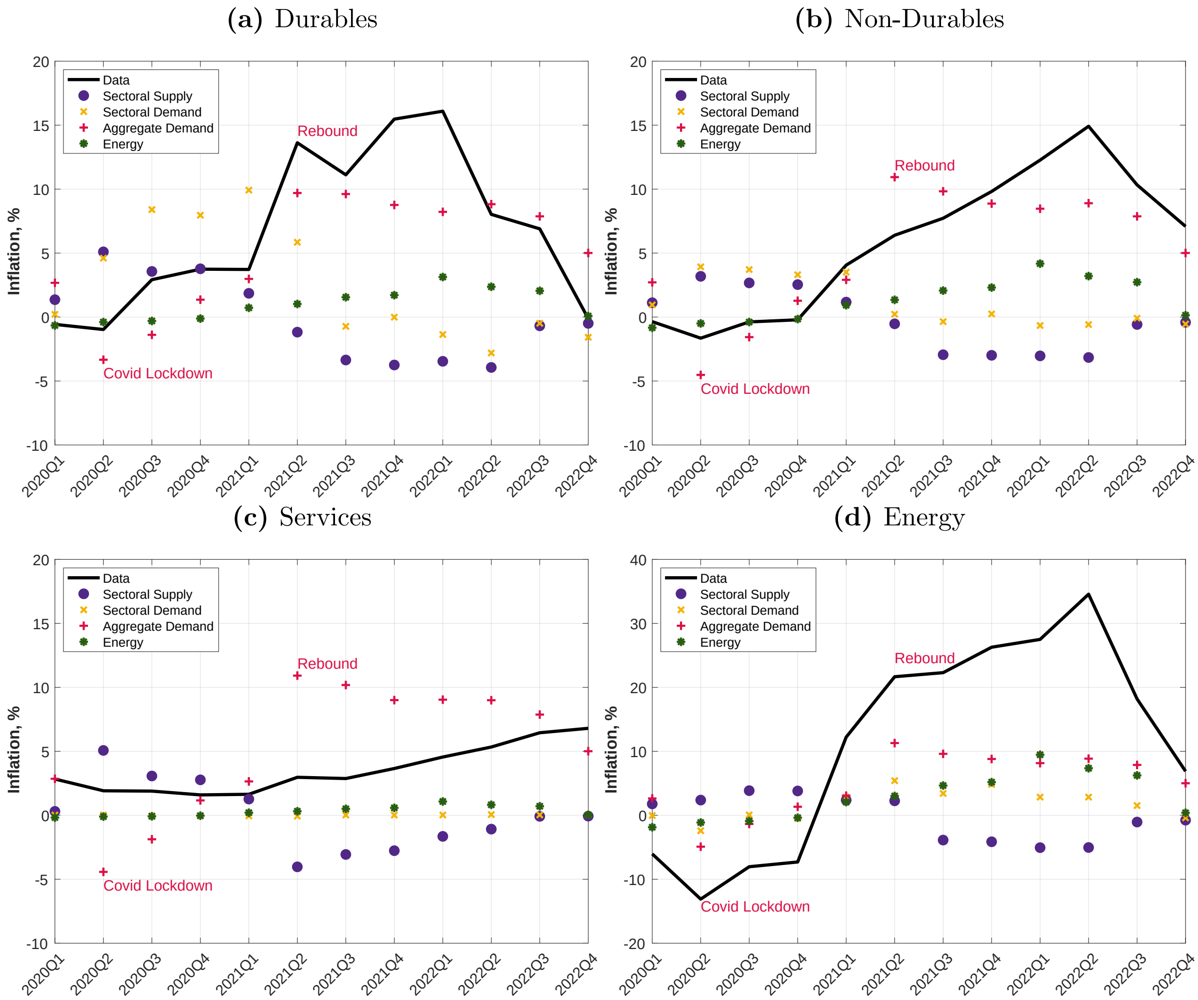


(b) Services



Note: This figure shows nominal wage growth (year on year) implied by the baseline model (blue diamonds) relative to the data (black lines).

Figure B.3. Sector-level Prices United States: Decomposition



Note: This figure shows annual inflation implied by the baseline model (green $*$) relative to the model with high elasticities of substitution (pink diamonds).

C Proofs

Proof of Proposition 1

The rich structure that we introduced in our model can be simplified to capture the first-order effect of shocks on inflation. Here, we will just focus on factors, goods and consumption ignoring the bundling at different levels. Starting from Equation (3), we can write production in terms of all other sectors and factors as:

$$Y_{ni} = A_{ni} F_{ni} \left(\{X_{ni,mj}\}_{mj \in \mathcal{S}}, L_{ni}, K_{ni} \right),$$

where F_{ni} is a nested CES function, $\mathcal{S}$ is the set of all country-sector pairs and $X_{ni,mj}$ denotes the amount of output of country-sector mj used by ni. With this, we can write the firm profit maximization problem as:

$$\pi_{ni} = P_{ni} Y_{ni} - \sum_{mj \in \mathcal{S}} P_{mj} X_{ni,mj} - W_{ni} L_{ni} - R_{ni} K_{ni}.$$

Using Shepard's Lemma, we can write the change in prices in terms of price changes of all other sectors and factor price changes as:

$$d \log P_{ni} = -d \log A_{ni} + \sum_{mj \in \mathcal{S}} \frac{P_{mj} X_{ni,mj}}{P_{ni} Y_{ni}} d \log P_{mj} + \frac{W_{ni} L_{ni}}{P_{ni} Y_{ni}} d \log W_{ni} + \frac{R_{ni} K_{ni}}{P_{ni} Y_{ni}} d \log R_{ni}.$$

Recall that:

$$X_{ni,mj} = X_{n,mj} \frac{X_{ni,j}}{X_{n,j}}.$$

At steady state, define the country-sector to country-sector input-output matrix as:

$$\begin{aligned}
\Omega^{SS}_{ni,mj} \equiv \frac{P_{mj} X_{ni,mj}}{P_{ni} Y_{ni}} &= \frac{P_{mj} X_{n,mj}}{P_{ni} Y_{ni}} \frac{P_{n,j} X_{ni,j}}{P_{n,j} X_{n,j}} \\
&= \left(\frac{P_{mj} X_{n,mj}}{P_{n,j} X_{n,j}} \right) \left(\frac{P_{n,j} X_{ni,j}}{P^Z_{ni} Z_{ni}} \right) \left(\frac{P^Z_{ni} Z_{ni}}{P_{ni} Y_{ni}} \right) \\
&= \Omega^X_{n,mj} \, \Omega^Z_{ni,j} \, \Omega^Y_{ni,Z}.
\end{aligned}$$

Similarly, we write the labor and capital shares as:

$$\Omega_{ni}^{SF,L} \equiv \frac{W_{ni}L_{ni}}{P_{ni}Y_{ni}} = \left(\frac{W_{ni}L_{ni}}{P_{ni}^{\text{VA}}\text{VA}_{ni}}\right)\left(\frac{P_{ni}^{\text{VA}}\text{VA}_{ni}}{P_{ni}Y_{ni}}\right) = \Omega_{ni,L}^{\text{VA}}\,\Omega_{ni,\text{VA}}^{Y},$$
$$\Omega_{ni}^{SF,K} \equiv \frac{R_{ni}K_{ni}}{P_{ni}Y_{ni}} = \Omega_{ni,K}^{\text{VA}}\,\Omega_{ni,\text{VA}}^{Y}.$$

Finally, the consumption share of country-sector mj is expressed as:

$$\Omega_{n,mj}^{CS} \equiv \frac{P_{mj}C_{n,mj}}{P_nC_n} = \left(\frac{P_{mj}C_{n,mj}}{P_{n,j}^{CB}C_{n,j}}\right)\left(\frac{P_{n,j}^{CB}C_{n,j}}{P_nC_n}\right) = \Omega_{n,mj}^{CB}\,\Omega_{n,j}^{C}.$$

With these definitions, we can write the changes in prices in vector notation with (and combining capital:

$$d\log P = -d\log A + \Omega^{SS}d\log P + \Omega^{SF,L}d\log W + \Omega^{SF,K}d\log R.$$

Defining the Leontief inverse for Ω^{SS}:

$$\Psi^{SS} = \left[I - \Omega^{SS}\right]^{-1},$$

we can solve for the price changes in terms of productivity change and factor price changes:

$$d\log P = -\Psi^{SS}d\log A + \Psi^{SS}\Omega^{SF,L}d\log W - \Psi^{SS}\Omega^{SF,K}d\log R.$$

Similarly, the CPI can be written as the weighted average of the good prices with weights $\Omega_{n,mj}^{CS}$. With this, the CPI can be written as:

$$d\log \text{CPI}_n = \sum_{mj}\Omega_{n,mj}^{CS}d\log P_{mj} = \Omega_n^{CS}d\log P,$$

where Ω_n^{CS} is the n^{th} row of the Ω^{CS} matrix. Combining with the price change equation, we can write the CPI change as:

$$d\log \text{CPI}_n = -\Omega_n^{CS}\Psi^{SS}d\log A + \Omega_n^{CS}\Psi^{SS}\Omega^{SF,L}d\log W - \Omega_n^{CS}\Psi^{SS}\Omega^{SF,K}d\log R.$$

Let's define the country-specific Domar weight for the labor:

$$(\Lambda^n)^T \equiv \Omega_n^{CS} \Psi^{SS} \Omega^{SF,L}$$

as the share of expenditures of country n that ends up in the owners of L_{mj}. We can write a similar expression for capital with

$$(\kappa^n)^T \equiv \Omega_n^{CS} \Psi^{SS} \Omega^{SF,K}.$$

Since the factors are where all the payments are accumulated, sum over these Domar weights equal to 1:

$$\sum_{mj} \Lambda_{mj}^n + \kappa_{mj}^n = (\Lambda^n)^T \, 1_{\mathcal{JN}} + (\kappa^n)^T \, 1_{\mathcal{JN}} = 1,$$

where $1_{\mathcal{JN}}$ is a vector of ones of a dimension $\mathcal{JN}$. Similarly, we can define the country-specific sector Domar-weights as:

$$(\lambda_n)^T = \Omega_n^{CS} \Psi^{SS}.$$

Hence, the CPI can be written as:

$$d \log \mathrm{CPI}_n = -(\lambda_n)^T d \log A + (\Lambda^n)^T d \log W - (\kappa^n)^T d \log R.$$

The Domar weights for labor and capital also satisfy:

$$\Lambda_{mj}^n = \frac{W_{mj} L_{mj}^n}{I^n},$$

where L_{mj}^n is the portion of the labor in mi that could be attributed to country n. Hence, we can write the change in wages in terms of their Domar weights as:

$$d \log W_{mj} = d \log \Lambda_{mj}^n - d \log L_{mj}^n + d \log I^n = d \log \Lambda_{mj}^n - d \log L_{mj} + d \log I^n.$$

In the last equality, we made the assumption that the labor change reduces proportionately regardless of the destination. Therefore, we can drop the country superscript. For the rental rate:

$$d \log R_{mj} = d \log \kappa_{mj}^n - d \log K_{mj}^n + d \log I^n = d \log \kappa_{mj}^n + d \log I^n,$$

since capital levels are assumed to be constant. With these, we can write the CPI as:

$$\begin{aligned}
d \log \mathrm{CPI}_n &= -(\lambda_n)^T d \log A + (\Lambda^n)^T d \log \Lambda^n - (\Lambda^n)^T d \log L^n \\
&+ \underbrace{\left((\Lambda^n)^T 1_{\mathcal{JN}} + (\kappa^n)^T 1_{\mathcal{JN}}\right)}_{=1} d \log I^n + (\kappa^n)^T d \log \kappa^n \\
&= -(\lambda_n)^T d \log A + \underbrace{(1_{\mathcal{JN}})^T d\Lambda^n + (1_{\mathcal{JN}})^T d\kappa^n}_{d[(1_{\mathcal{JN}})^T \Lambda^n + (1_{\mathcal{JN}})^T \kappa^n] = d(1) = 0} - (\Lambda^n)^T d \log L^n + d \log I^n \\
&= d \log I^n - (\Lambda^n)^T d \log L^n - (\lambda_n)^T d \log A,
\end{aligned}$$

where in the second equality we used the fact that: $L_{mj} \, d \log L_{mj} = d \, L_{mj}$. $\square$