Business Cycle Asymmetry and Input-Output Structure: The Role of Firm-to-Firm Networks*

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Abstract

We study the network origins of business cycle asymmetries using cross-country and administrative firm-level data. At the country level, we document that countries with a larger number of non-zero intersectoral linkages (denser networks) display a more negatively skewed cyclical component of output. At the firm level, we find that firms with a larger number of suppliers and customers (degrees) display a more negatively-skewed distribution of their output growth. To rationalize these findings, we construct a multisector model with input-output linkages and show that the relationship between output skewness and network density naturally arises once we consider non-linearities in production. In an economy with low production flexibility (inputs are gross complements), denser production structures imply that relying on more inputs becomes a risk that further amplifies the effects of negative productivity shocks. The opposite holds if firms display high production flexibility (inputs are gross substitutes): having more inputs to choose from becomes an opportunity to diversify the effects of negative productivity shocks. We calibrate the model using our rich firm-to-firm network Chilean data and show that more connected firms experience larger declines in output in response to a COVID-19 shock, consistent with the data. The size of the shock determines the strength of the relationship between degrees and output decline, which highlights the importance of non-linearities and the limitations of local approximations.

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1 Introduction

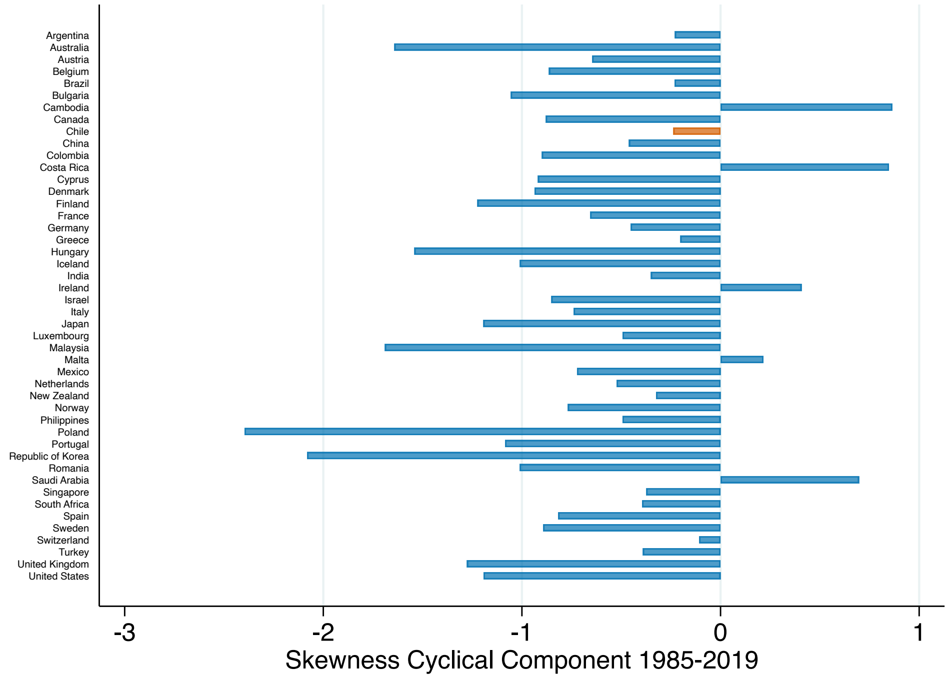
It is a general fact that recessions are shorter and more severe than expansions, i.e. they are "sharper". This asymmetry leads to a negatively-skewed distribution of real GDP growth as documented in, for example, Ordonez (2013). Figure 1 shows this asymmetry for the cyclical component of real GDP for a sample of 46 countries during 1985-2019. Out of the 46 countries, 43 display negatively-skewed business cycles. The primary explanation for this fact in the literature is the existence of financial constraints (Ordonez, 2013; Jensen et al., 2020). In this paper, we offer a different explanation for the asymmetry based on the empirical importance of sectoral shocks and the structure of input-output connections.

We use sectoral input-output data for 46 countries and firm-to-firm network data for the Chilean economy to study the role of production networks in shaping the magnitude of macroeconomic downturns. In the cross-country data, we document strong correlations between the skewness of the cyclical component of real GDP — our measure of the downturn's severity—and input-output structure (density of the network). We then use the firm-level network data to investigate to which extent production linkages at the firm level relate to firm-level output growth skewness and firm-level output declines during COVID-19.

Using OECD domestic input-output data, we show that — controlling for other important cross-country characteristics — countries in which more input-output connections are active (denser networks) display more negatively-skewed business cycles, as expressed by a more negative skewness of the cyclical component of real GDP for the period 1985-2019. Our estimates imply that if a country with a network density of 0.69 (the average in the sample) were to increase its active links by 10 percentage points (to 0.79), the skewness of the cyclical component of real GDP would decrease from -0.68 to -0.98. To put the numbers into perspective, a country with a skewness of -0.68 (e.g., Italy) experiences an average percent decline in real GDP of -2.3 percent, while a country with a skewness of -0.98 (e.g., Portugal) displays an average downturn of -3.5 percent.

We then use administrative data on Chilean firm-to-firm transactions to investigate the relationship between firm-level interconnectedness, as described by the total degrees of the firm—defined as the total number of suppliers and customers the firm has—and firm-level resilience to negative shocks. In particular, we measure firm-level output (sales and employment) growth skewness and show that, controlling for covariates, firms with a larger number of suppliers and customers display a more negatively skewed distribution of out-

Figure 1. Skewness of Cyclical Component of Real GDP (1985 – 2019)



Note: This figure plots the skewness of countries' annual cyclical component of real GDP for the period 1985-2019. We compute the cyclical component following Hamilton (2018) and estimate it as the residual (ε_t) from a regression of the form $y_t = \beta_0 + \beta_1 y_{t-2} + \beta_2 y_{t-3} + \varepsilon_t$ country-by-country, where y_t is log real GDP at time t.

put growth.[1] This relationship hides an important asymmetry as the negative relationship is mainly driven by the firms with negative skewness. The group of firms with positive skewness instead displays a positive, albeit smaller, relationship between degrees and output skewness. We then study the performance of more interconnected firms during COVID-19. We show that, controlling for other firm-level covariates, firms with more connections during COVID-19 experienced larger declines in sales and employment in 2020q2. We also find that during the recovery period, more connected firms were able to grow slightly faster than less connected firms.

We explain our evidence and quantify the role of networks in a production network model with N firms connected through intermediate input purchases. Our approach follows Baqaee and Farhi (2019) closely, in which non-linearities in production can generate asymmetric business cycles out of symmetric idiosyncratic technology shocks. We start by extending

[1] Our baseline specification uses the sum of buyers and suppliers as our preferred measure of firms' interconnectedness. We also studied the relationship between output skewness and the number of suppliers (indegree) or buyers (outdegree) separately. Our results are robust to use the sum of these measures or each one separately. See Section 3.3 for more details.

the analysis in Baqaee and Farhi (2019) and show the role of network density—the number of active input-output links—in amplifying or mitigating negative productivity shocks.[2] To do so, we study two networks that only differ in the number of positive input-output links and show that more connections amplify the adverse effects of negative productivity shocks, creating a more negative skewness of output if inputs are gross complements; the opposite holds if firms have a higher flexibility in substituting their inputs. The intuition behind our results lies in the strength of sectoral/firm-level price and quantity adjustments. In our model, price adjustments depend only on the network structure to a first order. Quantity adjustments, also to a first order, depend on the network structure and on the flexibility in substituting inputs. If inputs are gross complements, a negative productivity shock in the dense network generates a Baumol-cost disease mechanism in which the sector hit by the negative shock becomes larger in the economy relative to a more sparse network.

We then perform two quantitative exercises to understand the drivers of macroeconomic skewness and firm-level responses to negative productivity shocks. First, we calibrate our model economy to match the production network of the 46 countries in our sample. The model-implied skewness of log real GDP is negative in all countries and displays significant cross-country heterogeneity[3]. Moreover, we show that the model delivers a relationship between network density and skewness that is qualitatively similar to that in the data, although not quantitatively.

Our second quantitative exercise uses the firm-to-firm network structure of the Chilean economy before COVID-19. We investigate the ability of the model to deliver the non-linear relationship between firm-level (in and out) degrees (the number of customers and suppliers) and output growth before, during, and after COVID-19. We calibrate the decline in firm-level productivity using the annual percent change in revenue labor productivity. The model can deliver a relationship between degrees and output growth that is very similar to that in the data. We show that the magnitude of the shock is crucial for delivering these facts and that conditional on the size of the shock, the relationship between degrees and output growth is stronger for negative productivity shocks than for positive ones. Therefore, the model's internal propagation is strong enough to deliver a procyclical cross-sectional skewness of output growth, even if productivity shocks are symmetric. Finally, the concavity of aggregate output in this economy reconciles two seemingly contradictory facts: at the firm level, about

[2] In our model, as in Carvalho et al. (2020), productivity shocks propagate upstream and downstream in the network. Therefore, indegrees and outdegrees both determine the exposure of firms to shocks.

[3] Note that 41 out of 46 countries in our sample features negative skewness of the cyclical component of real GDP.

half of the firms display positive skewness of output growth and half negative, while at the aggregate level, the economy has negatively-skewed output growth.

Contribution to the literature. The results in our paper highlight the risks of production interconnectedness when the economy is hit by large shocks. Our paper contributes to the literature that underlines the role of financial frictions in generating asymmetric business cycles (e.g., Ordonez (2013) and Jensen et al. (2020)). We instead propose a mechanism that relies on production non-linearities and the structure of the production network.[4]

Our paper also contributes to the literature that studies the role of production network density in determining the level of GDP (Herskovic, 2018), GDP growth (Acemoglu and Azar, 2020), and the volatility of GDP (Miranda-Pinto, 2021). Unlike the previous papers, we focus on the non-linear effects of large negative shocks to productivity and the role of firm-to-firm networks. Our paper is more closely related to Baqaee and Farhi (2019) and Dew-Becker (2022). Similar to Dew-Becker (2022), we complement Baqaee and Farhi (2019) by focusing on a particular network statistic (network density) and on higher-order moments of GDP. Compared to these papers, our contribution is twofold. First, we provide empirical evidence both at the country level and at the firm level that highlights the role of production networks in driving business cycle asymmetries. Second, our firm-to-firm network data allow us to test the model's predictions in the cross-section at a more granular level during COVID-19. We show that the size of the shock is crucial to deliver the observed relationship between firm-level degrees and economic resilience during COVID-19, as measured by the firm-level decline in output. Therefore, we highlight the importance of non-linearities and the need for global methods (or higher-order approximations) to solve models of intersectoral linkages and CES production technologies.

Our paper also connects to Salgado et al. (2019), who investigate the sources of the asymmetry in the cross-sectional distribution of firm-level output growth. We see our work as complementary to theirs. While they argue that shocks to the skewness of firm-level TFP shocks are important in recessions, we instead emphasize the non-linear role of firm-level network structures in amplifying (large) negative productivity shocks.

[4]Our mechanism is likely to be amplified by the presence of financial frictions. In an environment like the one developed by Bigio and La'O (2020), Miranda-Pinto and Zhang (2020) show that trade credit linkages can generate asymmetric effects of financial shocks along the supply chain.

2 Cross-country evidence

In this section, we analyze the cross-sectional relationship between countries' cyclical component of real GDP asymmetry, as measured in Figure 1, and countries' input-output structure. We collect data on real GDP (domestic currency) for the period 1985-2019 and real GDP per capita (at chained PPPs in 2017 US dollars) in 1985 from the Penn World Tables version 9.0. To measure the details of the production network structure across countries, we use the OECD input-output database. This dataset contains input-output data for about 60 countries at a level of disaggregation of 45 sectors for the period 1995-2018. Our final sample has 46 countries, of which about half are developed and half are emerging countries. We use the input-output data from 1995, which is the earliest available in the OECD database.

Figure 2 depicts the input-output network for Chile. In this figure, an arrow from sector j to sector i represents intermediate inputs flowing from j to i. The size of the nodes is determined by the size of the sector, in terms of gross output. The figure highlights the network features we focus on in this paper: the significant heterogeneity in sectoral degrees (number of suppliers and clients) and the level of production interconnectedness in the economy. We measure interconnectedness using network density as in, for example, Miranda-Pinto (2021). In particular, our measure of density is

$$\text{Density} = \frac{\sum_{i=1}^{N} \sum_{i=j}^{N} \mathbf{1}(\omega_{ij} > 0) - N}{N(N-1)};$$

where the numerator counts the number of non-zero off-diagonal input-output links in the economy, and the denominator sums all the possible off-diagonal input-output connections in the economy. Thus, density measures the fraction of feasible connections that are active in an economy. As an example, the Chilean production network displays a density of 69%.[5]

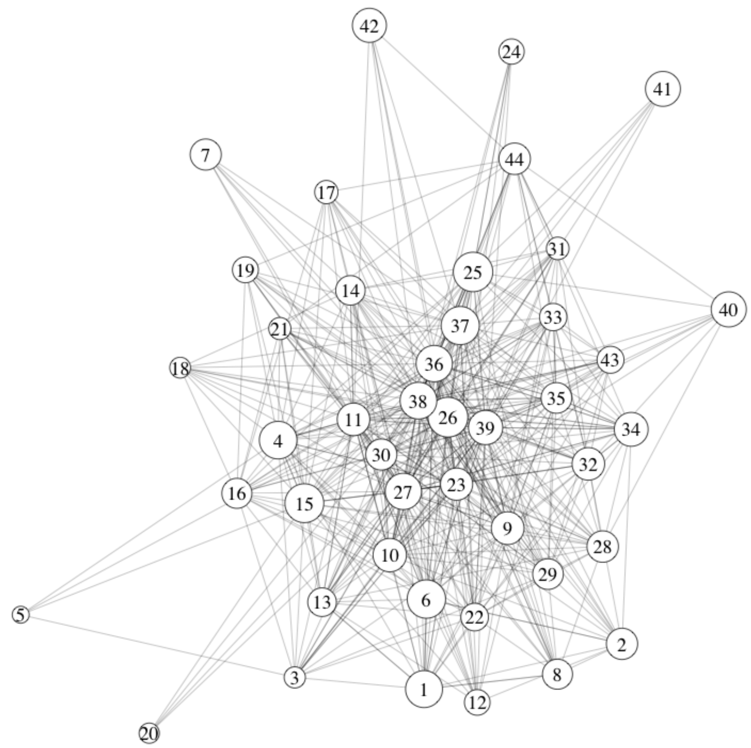
Our focus in this section is the cross-country correlation between production network density and the skewness of the cyclical component of real GDP. We measure the cyclical component of real GDP following Hamilton (2018), which hereafter we will call the Hamilton filter. We run the following regression country-by-country:

$$y_t = \beta_0 + \beta_1 y_{t-2} + \beta_2 y_{t-3} + \varepsilon_t,$$

where y_t is the log of real GDP at time t and ε_t is our measure of the cyclical component.

[5] We use a very small threshold to define a non-zero input-output link. In particular, we use a value of 0.1% for the ratio between a specific intermediate input expenditure and total intermediate input expenditure by each sectoral pair.

Figure 2. Input-Output Structure: Chile in 1995



Note: This figure shows the structure of inter-sectoral linkages for the Chilean economy in 1995 using the 45 sectors classification in the OECD input-output data revision 4. A node is a sector and the size of the node depends on the sectoral gross output.

As discussed in Appendix A, we chose the Hamilton filter over linear detrending or the Hodrick-Prescott filter because it better captures downturns in the data, a must for our exercise.

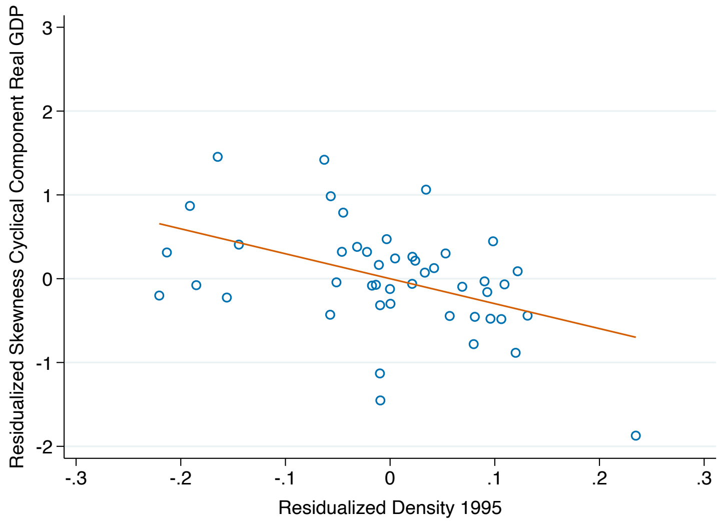
To measure the cross-country correlation between density and skewness, we control for other possible network moments such as the mean, standard deviation, and skewness of the weighted outdegrees and indegrees.[6] In addition, we control for other development measures such as the GDP per capita in 1985 and the volatility of the cyclical component during the period.

Figure 3 shows a scatter plot of the residual skewness of the cyclical component of real GDP and network density after removing any variation coming from our controls. As is apparent from this figure, the cross-country correlation between the cyclical component's

[6]The *weighted outdegree* of a sector i is the sum of the shares of sector i sales to sector j as a fraction of sector j's total output, for all j. The *weighted in-degree* of a sector i is the sum of the shares of sector i's purchases of intermediate inputs from sector j as a fraction of sector i's total output, for all j.

skewness and density is strongly negative, meaning that countries with more connections feature on average lower skewness of their cyclical component of real GDP. Since, on average, countries exhibit negative skewness of their cyclical component, this result means that countries with more interconnected production networks tend to exhibit more negatively skewed business cycles.

Figure 3. Cross-Country Production Network Density and Skewness



Note: This figure plots the residualized skewness of the cyclical component of real GDP between 1985 – 2019 and the residualized network density in 1995. These are residualized using the following controls: GDP per capita in 1985, the volatility of the cyclical component of real GDP, skewness of the weighted outdegree, and the weighted indegree distribution in 1995. See Table B.1 in the Appendix for magnitudes of the correlation between the two variables.

3 Firm-level evidence

In this section, we present evidence that relates firm-level output (sales and employment) growth asymmetry to firm-level network structures. We start by describing our data and then perform two empirical exercises. First, we study the relationship between firm-level networks and firm-level output growth skewness. Second, we investigate the relationship between firm-level networks and output growth during COVID-19.

3.1 Data Description and Sample construction

We combine four different administrative datasets collected by the Chilean tax authority (Servicio de Impuestos Internos, SII) that provide detailed firm-level information based on firms' tax ID numbers anonymized for research purposes.[7] [8] We use data on firms' total sales from the monthly and annual tax declarations for the period 2005-2020. In particular, the F29 form keeps track of monthly sales, while the F22 form has final information on annual sales for the tax payments. We also use firm-level information on employment from the form DJ1887. Finally, we use data at the firm-to-firm transaction level from the electronic transaction system implemented since 2014.[9] These data cover the universe of formal firms in Chile, and reporting is mandatory for all firms since mid 2018.

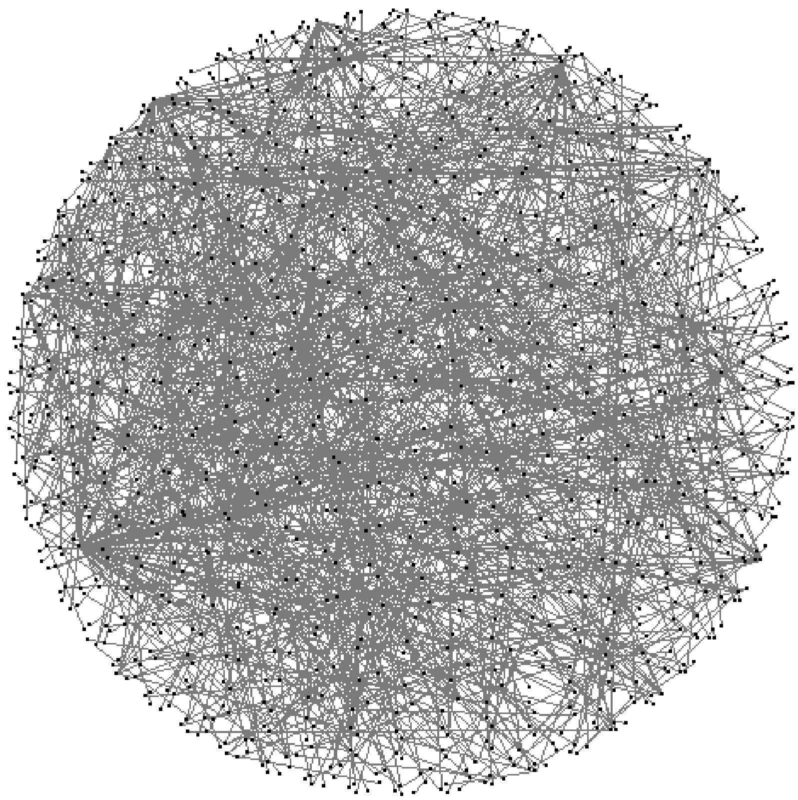
We combine the monthly sales data in F29 with the annual sales data in F22 because the high-frequency data in F29 is more likely to suffer from misreporting problems. For example, firms could misreport sales in May but report sales in June as sales from May and June. On the other hand, the F22 annual form, which is the official data for tax purposes, should be less susceptible to misreporting, both because it is also an official document and because it is at a lower frequency. Therefore, we use the high-frequency sales data only for firms whose sales reported in the F29 represent between $90 - 110$ percent of the sales reported in the F22. In addition, we drop the bottom and top 1 percent of the observations in terms of sales growth. For this group of firms, we also obtain their total employment data from the

[7]This study was developed within the scope of the research agenda conducted by the Central Bank of Chile (CBC) in the economic and financial affairs of its competence. The CBC has access to anonymized information from various public and private entities by virtue of collaboration agreements signed with these institutions.

[8]The information contained in the databases of the Chilean IRS is of a tax nature originating in self-declarations of taxpayers presented to the Service; therefore, the veracity of the data is not the responsibility of the Service.

[9]To secure the privacy of workers and firms, the CBC mandates that the development, extraction, and publication of the results should not allow the identification, directly or indirectly, of natural or legal persons. All the analysis was implemented by the authors and did not involve nor compromise the IRS.

Figure 4. Chilean Firm-to-Firm Network: Random Sample of 2000 firms



Note: This figure plots the firm-to-firm network in 2019q4, for a random sample of 2000 Chilean firms. A dot represents a firm, and each edge is an intermediate input sale that represents at least 10% of the client's total intermediate input purchases.

DJ1887 tax declaration form, which requires firms to report their wage bill and the number of employees. We then aggregate these monthly data to quarterly for the period 2005q1-2020q4. For firms' sales, we simply add sales across months in a given quarter. For employment, we aggregate it by taking the simple average across months in a given quarter. For the firm-to-firm transaction data, we add up all sales for a given pair of firms (i, j) across all months in a given quarter. As a final filter, we keep firms with five or more employees that are present at least 20 quarters for the period 2005q1-2020q4. To visualize how detailed our firm-to-firm data is, Figure 4 plots a random sample of 2000 firms in 2019q4. For visualization purposes, we plot the links representing at least 10% of firms' total intermediate input purchases.

From the firm-to-firm transaction data used to construct Figure 4 we measure the unweighted indegree of firm i as the number of firms that supply a positive amount of goods or services to firm i, while we measure the unweighted outdegree of firm i as the number of firms that buy a positive amount of the output produced by firm i. We calculate the total degrees as the sum of indegrees and outdegrees at the firm level. This *total degree* is our preferred measure for assessing how many connections each firm has throughout the paper, and henceforth, we will call it *degree*. Although there are compelling reasons for using either the indegree or outdegree as the correct measure of connectedness for a given firm, it turns out that both play a role in shaping the firm-level responses we focus on — sales and employment — as we discuss in the Section 4.

We also consider different moments of the distribution of firms' weighted indegrees and outdegrees, which we use as controls in our regressions and to calibrate our quantitative exercises below. In particular, we obtain the average indegree as the ratio between firm i's total expenditure on other firms' output and firm i's total sales $\Omega_{iM} = \frac{\sum_{j=1}^{N} P_j M_{ij}}{P_i Q_i}$. We then obtain the input-output shares as $\Omega_{ij} = \frac{P_j M_{ij}}{P_i Q_i}$.[10]

Table 1 presents the main descriptive statistics. We report average sales, the average number of employees, degrees, volatility, and skewness of output growth. Two main facts stand out. First, network degrees display significant heterogeneity. Second, about half of the firms in our sample display a negatively skewed distribution of output growth.

Table 1. Descriptive statistics

	Mean	SD	p25	p50	p75	Obs.
Sales 1st period (millions)	314.290	4,770.680	12.186	34.507	96.921	68,885
Average employees	40.330	189.481	6.486	11.400	24.564	68,885
Degree 1st period	40.654	64.748	10.000	20.000	41.000	68,885
Standard deviation sales growth	0.500	0.282	0.278	0.438	0.676	68,885
Standard deviation employment growth	0.405	0.308	0.192	0.313	0.521	68,875
Skewness sales growth	-0.103	0.966	-0.489	-0.046	0.374	68,885
Skewness employment growth	0.048	1.230	-0.487	0.066	0.629	68,855

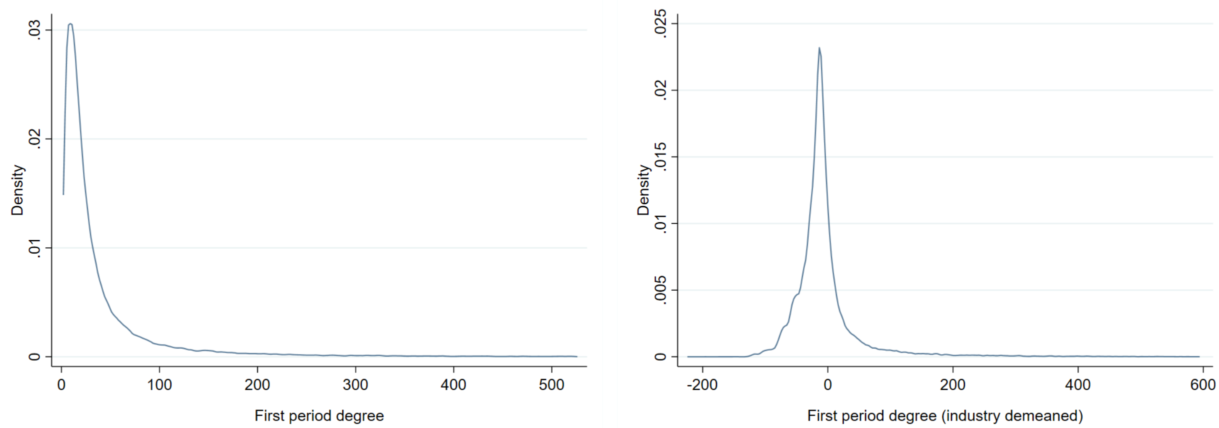
Note: This table presents basic descriptive statistics on output, network, volatility of output growth, and skewness of output growth. Sales first period and degree first period correspond to the sales and degrees that firms display either at the beginning of the period in 2005q1 or whenever the firm enters the sample (as long as it meets the requirement of 20 quarters in the sample).

[10]We also follow an alternative approach to measure firm-level linkages. In particular, instead of using firm-to-firm linkages, we use disaggregated industry classifications (170 industries) to measure degrees at the firm-to-industry level. The advantage of this approach is that it can better describe different intermediate inputs in the production process (e.g., metal vs glass) rather than different varieties of the same intermediate input (e.g., glass type A and glass type B that differ little and are simply sold by competitor firms). The results are very similar, which is why we prefer to use firm-to-firm linkages throughout the paper.

3.2 Firm-level networks

Here, we provide more information on the distribution of firm-level degrees. Figure 5, panel a, shows that degrees have a thick right tail. A relatively small number of firms are very well connected. Indeed, the average degree is twice as large as the median degree. Similar results hold when we consider the average degree over the period.

Figure 5. Network degrees distribution



Note: This figure presents the kernel distribution of firm-level degrees the first time these firms report data on linkages. Panel (a) plots the distribution of the raw data, while panel (b) reports the firm degree subtracting the industry average (170 industries classification).

Our firm-level network data allows us to study the heterogeneity in linkages within narrowly-defined industries. Panel (b) of Figure 5 shows that there is substantial heterogeneity in degrees across firms, even after removing industry-fixed effects. Consider the following two examples: bakery products and hotels. There are $1,509$ companies in the bakery products industry with an average first-period degree of 26.9 links, a standard deviation of 32.8, and a skewness of 7.2. In comparison, in the hotels industry there are $1,376$ companies with an average first-period degree of 43.9 links, a standard deviation, and a skewness of 58.7 and 3.7, respectively.

We now study the relationship between firm size and linkages in the cross-section. To do so, we run the following regression

$$\log \text{Degree}_i = \alpha + \alpha_I + \beta_s \log \text{Sales}_i + \boldsymbol{\gamma}' \boldsymbol{X}_i + \varepsilon_i, \tag{1}$$

where $\log \text{Degree}_i$ is the average number of degrees (number of customers and suppliers) that a given firm i has during our analyzed period. α is a constant term, α_I is an industry fixed-effect, and $\boldsymbol{X}_i$ contains firm-level controls such as the average intermediate input and

Table 2. Size and Interconnectedness

	Dep. Var: log Degree		
	(1)	(2)	(3)
log Sales	0.333***	0.338***	0.301***
	(0.003)	(0.003)	(0.003)
Observations	64,642	64,642	64,642
R-squared	0.250	0.251	0.386
Controls	No	Yes	Yes
Sector FE	No	No	Yes

Note: This table reports the OLS coefficient of a regression in which the dependent variable is log degrees and the independent variable is log sales. Controls include average intermediate input share and the average export share (as a share of sales). Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

export share (as a share of total sales). ε_i is an error term. The parameter of interest is β_s, which is the elasticity of degrees with respect to changes in sales.

Unsurprisingly, in Table 2 we find that larger firms have more connections. In the cross-section, the elasticity of indegree with respect to sales is 0.30, meaning that moving up 1 percent in the distribution of firm size is associated with a 0.3 percent increase in the degree. The R^2 of the regression is 0.25, and once we control for other firm-level characteristics and sector fixed effects the R^2 of the regression only increases to 0.38. The positive relationship between degrees and size is consistent with the implications of multisector models with intersectoral linkages, as in Acemoglu et al. (2012). However, there is a significant portion of the variation in degrees (62 percent) that is not accounted for the cross-sectional variation in firm-level observables that we use. This result leads us to believe that the structure of firm-level linkages provides valuable information beyond the firm size and industry fixed-effects that we analyze in the next sections.

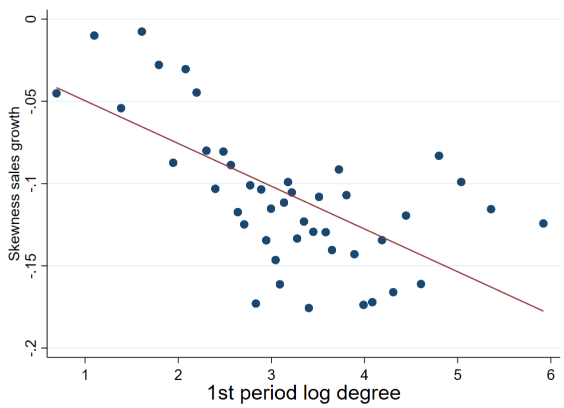
3.3 Firm-level output asymmetry and networks

In this subsection, we investigate the connection between firm-level asymmetry and network structure.

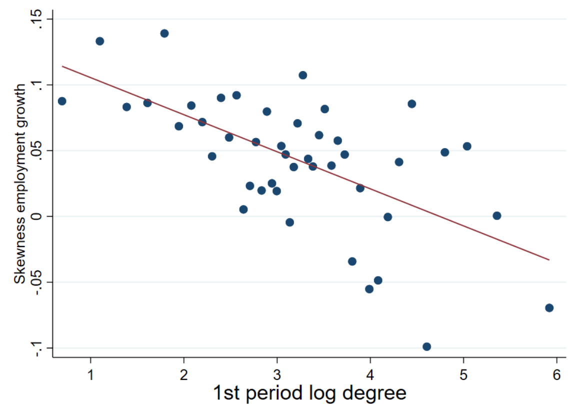
Figure 6 provides a first glance at the unconditional relationship between degrees and output growth skewness, measured using either sales (panel (a)) and employment (panel (b)). We observe a clear negative one: more interconnected firms display larger declines in

Figure 6. Output asymmetry and firm level networks

(a) Sales Growth Skewness



(b) Employment Growth Skewness



Note: These figures plot the binscatter plots, using 50 bins, for log degrees in the x-axis and the skewness of firm-level sales growth (panel a) the skewness of employment growth (panel b).

output than less connected firms.

We now study the relationship between firm-level networks and firms' asymmetry in output growth in more detail. To do so, we take advantage of the cross-sectional heterogeneity in firm-level output growth skewness for the whole sample and the average number of linkages (indegree, outdegree, and the sum of both). In particular, we estimate the following equation:

$$Y_i = \alpha + \alpha_I + \beta_d \log \text{Degree}_i + \boldsymbol{\gamma}' \boldsymbol{X}_i + \varepsilon_i, \tag{2}$$

where Y_i may represent either sales growth skewness or employment growth skewness during the period for a given firm i. Our parameter of interest is β_d which provides the relationship between log degree and the two skewness measures. We measure the degree of a firm using the average number of connections a firm has during the period, including both buyer and seller relationships.[11] The parameter α_I represents industry fixed effects that we include to account for the fact that some industries might be naturally exposed to more skewed shocks. The $\boldsymbol{X}_i$ represents firm-level characteristics and include size (in terms of sales), export share, and intermediate input share. Finally, ε_i is an error term.

[11]We also studied the relationship between output skewness and the number of suppliers (indegree) or buyers (outdegree) for each firm separately. The results indicate that each degree displays a negative and statistically significant relationship with output skewness. Since they do not add information separately, we used the total degree to measure network interconnectedness. As we will see later, the model we present indeed displays a similar role for both the indegree and the outdegree, which is why the total degree provides enough information.

Table 3. Sales Growth Skewness and Network Degrees

	Dep. Var: Sales Growth Skewness					
	Negative Skewness			Positive Skewness		
	(1)	(2)	(3)	(4)	(5)	(6)
Log degree 1st	-0.091***	-0.056***	-0.048***	0.040***	0.040***	0.044***
	(0.004)	(0.005)	(0.005)	(0.003)	(0.004)	(0.004)
Observations	36,534	34,216	34,209	32,309	29,841	29,834
R-squared	0.016	0.031	0.129	0.005	0.043	0.072
Controls	No	Yes	Yes	No	Yes	Yes
Sector FE	No	No	Yes	No	No	Yes

Note: This table reports the OLS coefficient of a regression in which the dependent variable is the skewness of sales growth and the independent variable is log degree. Controls include log sales, intermediate input share, and export share. Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

In Tables 3 and 4 we report the results of estimating Equation 2 for both sales growth skewness and employment growth skewness, respectively. Also, we partition the number of firms into those that exhibit either negative or positive skewness and run the regressions for those subsamples. The results show that the negative relationship between degrees and skewness only holds for the group of firms with negative skewness. Columns (1) to (3) in Table 3 show that more interconnected firms display a more negative skewness of output (sales and employment) growth, even after controlling for firm-level characteristics and industry-fixed effects. In columns (4) to (6), we observe that the opposite holds for the group of positively skewed firms.

3.4 Firm-level resilience and networks during downturns

In this subsection, we follow Salgado et al. (2019) and study the asymmetry in the cross-sectional distribution of output growth and show how the firm-level network structure could provide insights into this asymmetry. Figure 7 shows that during macroeconomic downturns the distribution of firm-level output growth displays a fatter left tail, consistent with Salgado et al. (2019).[12]

We now zoom into the COVID-19 crisis. COVID-19 is the only recession for which we have firm-to-firm network data and COVID-19 represents an ideal event study for at least three reasons. First, COVID-19 is a very large shock, which is therefore more likely to activate non-linear effects. Second, COVID-19 is a sectoral productivity shock that had

[12] Similar results hold for the size-weighted distribution of output growth.

Table 4. Employment Growth Skewness and Network Degrees

	Dep. Var: Employment Growth Skewness					
	Negative Skewness			Positive Skewness		
	(1)	(2)	(3)	(4)	(5)	(6)
Log degree 1st	-0.073***	-0.070***	-0.050***	0.035***	0.069***	0.070***
	(0.005)	(0.006)	(0.006)	(0.004)	(0.005)	(0.005)
Observations	31,992	30,329	30,324	36,859	33,709	33,705
R-squared	0.007	0.032	0.094	0.002	0.014	0.036
Controls	No	Yes	Yes	No	Yes	Yes
Sector FE	No	No	Yes	No	No	Yes

Note: This table reports the OLS coefficient of a regression in which the dependent variable is the skewness of employment growth and the independent variable is log degree. Controls include log sales, intermediate input share, and export share. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

heterogeneous effects across industries (contact services vs non-contact goods and services) and firms (small vs large firms).[13] Third, during COVID-19 in Chile, financial conditions actually improved due to the implementation of several policies aimed at supporting the most-affected firms (see, Albagli et al., 2021, for detailed evidence). Hence, COVID-19 represents a situation where the complementary hypothesis of financial frictions driving business cycle asymmetries may be relatively muted. As we highlight in Section 5.2, we propose that the large negative productivity shocks induced by COVID-19 were amplified via production interconnectedness.

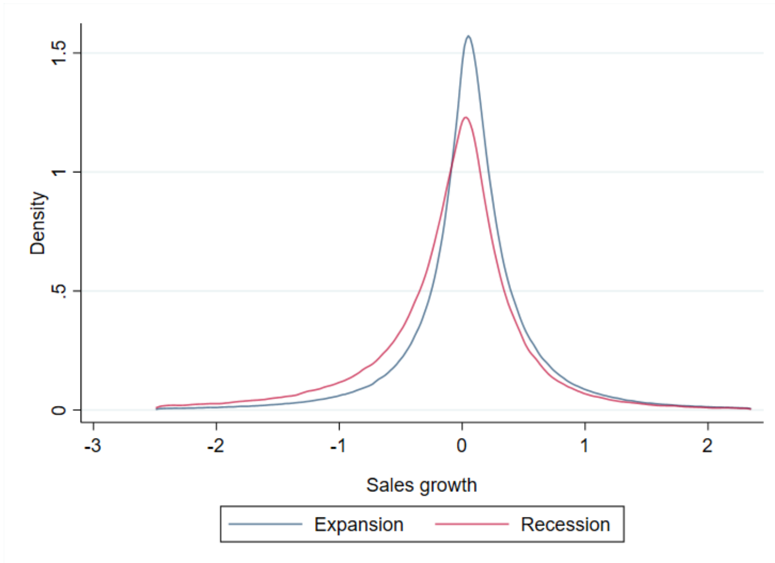
We show that, consistent with Figure 7, firm-level output growth skewness declined significantly during COVID-19: a significantly larger number of firms experienced declines in sales and employment. In particular, Figure 8 shows a large increase in the mass of firms in the left tail of the distribution of sales growth during 2020q2, compared to 2019q2. But who exactly were these firms? To answer this question, we run the following cross-sectional regression at each quarter t:

$$\Delta \log y_i(t) = \alpha(t) + \beta(t) \log degree_i^{2017q4} + \Gamma(t) \cdot controls(t) + \epsilon_i(t), \tag{3}$$

where $\beta(t)$ measures the importance of firm-level degrees in determining performance at time t compared to other firms. We include controls for industry fixed effects, sales, intermediate input shares, and export shares.

[13] There is an important demand component to the COVID-19 shock as well, due to lockdowns and other restrictions on business operations.

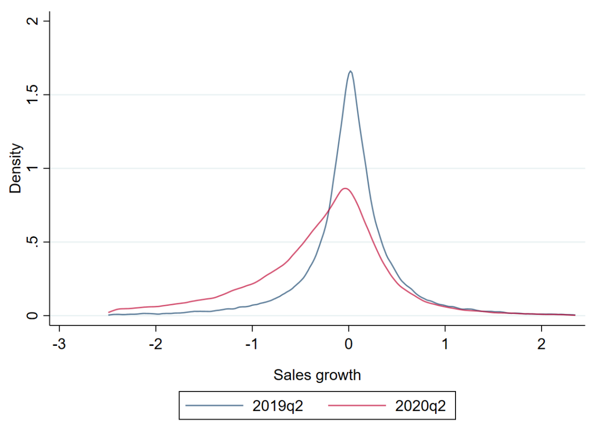
Figure 7. Output growth distribution recessions and expansions



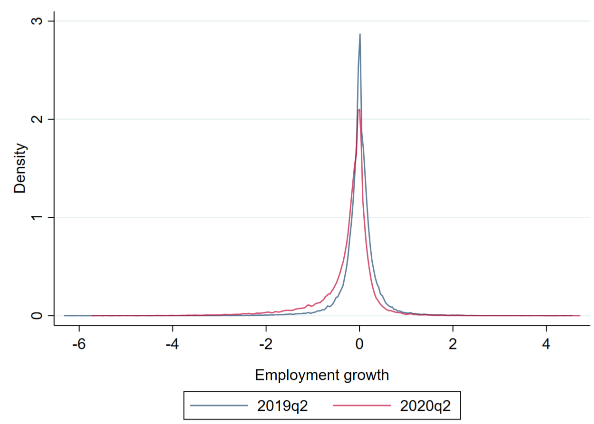
Note: This figure presents the kernel density of sales growth during expansions and recessions. Recessions are defined as the firm-quarter observations in 2009 (GFC) and 2020 (COVID-19), while expansions are all the other firm-quarter observations.

Figure 8. Output growth distribution during COVID-19

(a) Sales Growth



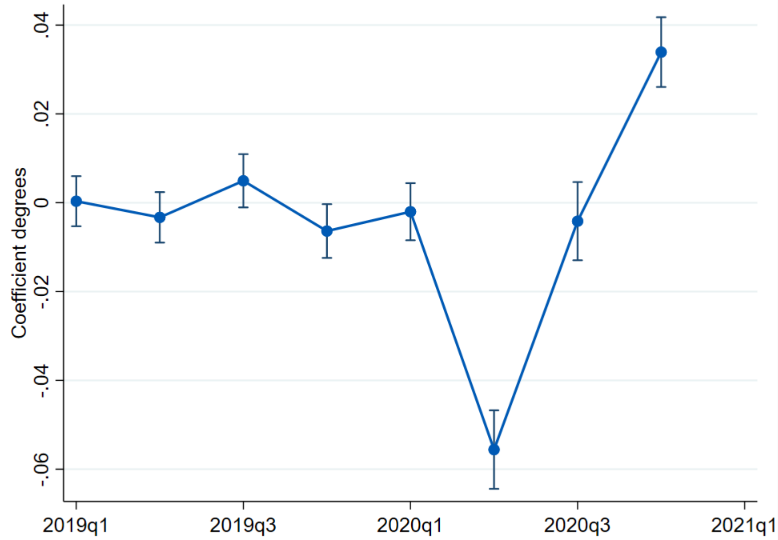
(b) Employment Growth



Note: This figure presents the kernel density of sales and employment growth before (2019q2) and during COVID-19 (2020q2).

In Figure 9 we plot the OLS-estimated coefficient $\beta(t)$ in Equation 3 at different quarters in 2019 and 2020. We can see that the structure of firm-level networks was a relevant predictor of firm performance during COVID-19, even after controlling for intermediate input share, export share, sales, weighted degree measures, and industry fixed effects. A firm with a degree 10% larger (4 extra links compared to the average in Table 1) displayed a

Figure 9. Coefficient of regressing output growth against log degrees



Note: This figure plots the OLS estimated $\beta(t)$ from Equation 3.

decline in output growth that was 0.55 percentage points larger. Interestingly, the coefficient of log sales is positive and equal to 0.04, indicating that a firm with sales 10% larger had 0.4 percentage points of larger (or less negative) sales growth. Hence, larger firms were more resilient to COVID-19, and, conditional on size, those that were more connected were less resilient. In the next section, we construct a production network model that helps us understand the cross-country and firm-level facts we document on the relationship between output skewness and network structure.

4 Theory

We consider a general equilibrium closed economy environment with a representative consumer, N producers, and F factors of production based on Baqaee and Farhi (2019). This economy features no distortions or frictions. We describe each block in turn.

Notation. Throughout, we use **bold** to denote vectors and matrices. For any vector/matrix $\boldsymbol{X}$, we use $\boldsymbol{X}^T$ for its transpose.

4.1 Representative Consumer

The representative consumer has preferences over the N different goods according to the utility function

$$U\left(C_1, C_2, ..., C_N\right) \tag{4}$$

where C_i represents consumption of good i. We assume $U\left(\cdot\right)$ is homothetic.

The representative consumer owns all F factors of production and supplies them inelastically to producers. Denoting the price of factor f as W_f, the representative consumer budget constraint is then

$$\sum_{i=1}^{N} P_i C_i \leq \sum_{f=1}^{F} W_f L_f + \sum_{i=1}^{N} \Pi_i \tag{5}$$

where Π_i is firm i's profit.

Consumer's problem. Taking as given good and factor prices $(\boldsymbol{P}, \boldsymbol{W})$, and factor supplies $\bar{\boldsymbol{L}}$ the representative consumer chooses a sequence $\boldsymbol{C}$ of consumption demands to maximize (4) subject to (5). The solution to this problem delivers *Marshallian demands* for each good i as a function of prices and factor supplies i.e. $C_i = C_i\left(\boldsymbol{P}, \boldsymbol{W}, \bar{\boldsymbol{L}}\right)$ for all $i = 1, 2, ..., N$. We denote the optimal vector by $\boldsymbol{C}^*$ and let $Y = U\left(\boldsymbol{C}^*\right)$ to be the maximum utility.

4.2 Producers

There are N producers indexed by $i = 1, 2, ..., N$. Each producer i produces quantity Q_i using factors $\{L_{if}\}_{f=1}^{F}$ and intermediate inputs from other producers $\{M_{ij}\}_{j=1}^{N}$. Here L_{if} represents the demand for factor f by producer i, and M_{ij} represents producer's i demand for good j. Each producer has access to a producer-specific production function that satisfies

$$Q_i = A_i F^i\left(\{L_{if}\}_{f=1}^{F}, \{M_{ij}\}_{j=1}^{N}\right) \tag{6}$$

where A_i is a Hicks-neutral technology level. We assume that the $F^i\left(\cdot\right)$ is constant returns to scale.

Given good and factor prices, the total cost of producer i is

$$TC_i = \sum_{f=1}^{F} W_f L_{if} + \sum_{j=1}^{N} P_j M_{ij}. \tag{7}$$

Producer's problem. Taking as given good and factor prices $(\boldsymbol{P}, \boldsymbol{W})$, each producer i minimizes (7) subject to (6). The solution to this problem delivers conditional demand for all inputs (both factors and intermediate goods), that are functions of prices, technology and quantities i.e. $L_{if} = L_{if}(P, W, Q_i, A_i)$ and $M_{ij} = M_{ij}(P, W, Q_i, A_i)$.

One consequence of our assumptions is that total costs can be written as

$$TC_i = TC_i(\boldsymbol{W}, \boldsymbol{P}, A_i, Q_i) = MC_i(\boldsymbol{W}, \boldsymbol{P}, A_i) Q_i, \tag{8}$$

so that total costs are linear in the quantity, Q_i. This result is a consequence of the constant returns to scale assumption.

By *Shephard's Lemma*, we can write conditional demands as

$$L_{if} = \frac{\partial TC_i}{\partial W_f} = \frac{\partial MC_i}{\partial W_f} Q_i \quad \text{for all } f = 1, 2, ..., F \tag{9}$$

$$M_{ij} = \frac{\partial TC_i}{\partial P_j} = \frac{\partial MC_i}{\partial P_j} Q_i \quad \text{for all } j = 1, 2, ..., N \tag{10}$$

4.3 Equilibrium

To close the model, we need to specify the market clearing conditions for both good and factor markets.

$$Q_i = C_i + \sum_{j=1}^{N} M_{ji} \quad \text{for all } i = 1, 2, ..., N \tag{11}$$

$$\bar{L}_f = \sum_{i=1}^{N} L_{if} \quad \text{for all } f = 1, 2, ..., F \tag{12}$$

Equation 11 are the goods market clearing conditions, while Equation 12 are the factor market clearing conditions.

4.4 Useful Definitions.

We now define some objects that are going to be key for our analysis.

We let $\boldsymbol{\Omega}$ to be the *input-output matrix* of this economy, with typical element

$$\boldsymbol{\Omega} = \{\Omega_{ij}\} = \frac{P_j M_{ij}}{P_i Q_i} \quad \text{for all } i, j = 1, ..., N$$

Typical element Ω_{ij} measures how much producer i spends on good j, $P_j M_{ij}$, as a fraction of i's sales, $P_i Q_i$. Here P_i is the price of good i, Q_i is the quantity sold of good i, and M_{ij} is how much producer i buys of the quantity of good j. In other words, it is a measure of the importance of producer j (column, seller) as a *supplier* to producer i (row, buyer).

With some abuse of notation, we also define the producer's i expenditure on factor f as a fraction of its sales

$$\Omega_{if} = \frac{W_f L_{if}}{P_i Q_i}.$$

We define $\mathbf{\Psi}$ as the *Leontief-Inverse matrix*, of dimension $N \times N$, as

$$\mathbf{\Psi} = (I - \mathbf{\Omega})^{-1} = \sum_{s=0}^{\infty} \mathbf{\Omega}^s \text{ with typical element } \{\Psi_{ij}\}. \tag{13}$$

Ψ_{ij} denotes how important is producer j as a *direct and indirect* supplier to producer i.

On the consumption side, we define the vector of consumption shares, $\boldsymbol{b}$, as

$$\boldsymbol{b} = \{b_i\} = \frac{P_i C_i}{GDP},$$

where C_i represents the consumption of good i.

Since there are F factors of production, we define their shares of *Nominal Gross Domestic Product* (GDP) as

$$\Lambda_f = \frac{W_f \bar{L}_f}{GDP}; \quad \sum_{f=1}^{F} \Lambda_f = 1; \quad \sum_{f=1}^{F} W_f \bar{L}_f = GDP,$$

where W_f is the price of factor f, $\bar{L}_f$ is the equilibrium factor f quantity, which in this model coincides with the factor supply endowment, $\bar{L}_f$. The second result is a restatement of the first result and follows from the fact that everything in this economy is produced out of factors. Therefore, the total value added (GDP) should equal factor payments.

We let λ_i denote the *Domar weight* of producer i in total value added:

$$\lambda_i = \frac{P_i Q_i}{GDP}.$$

In the presence of intermediate goods in production, the Domar weight is the relevant size statistic for each producer's contribution to total value added (Domar, 1961; Hulten, 1978).

4.5 Aggregate Impact of Sectoral Technology Shocks

We now focus on how the production network structure can affect aggregate output skewness. The following is a re-statement of a previous result in Baqaee and Farhi (2019):

Proposition 1 (Macroeconomic Impact of Sectoral Technology Shocks) *To a second-order approximation, around an equilibrium defined by output $\bar{Y}$ and a Domar weights vector $\bar{\boldsymbol{\lambda}}$ of size $N \times 1$, the macroeconomic effect of sectoral technology shocks on real GDP, Y_t, is given by*

$$\log Y_t = \log \bar{Y} + \sum_{i=1}^{N} \bar{\lambda}_i (\log A_{it} - \log \bar{A}_i) \left(1 + \frac{1}{2} \frac{d \log \bar{\lambda}_i}{d \log A_{it}} (\log A_{it} - \log \bar{A}_i)\right), \tag{14}$$

where we assume that sectoral technological changes are uncorrelated.

The above proposition highlights that sectoral technology shocks can have meaningful second-order effects on real GDP provided that the Domar weight $\bar{\lambda}_i$ reacts to changes in productivity $\log A_i$. How much $\bar{\lambda}_i$ reacts to the productivity shock depends on two key concepts: the elasticity of substitution of each producer and the production network structure. On the one hand, the elasticity of substitution is key for determining the sign of $d\bar{\lambda}_i$ (the direction of the response). The production network structure, on the other hand, provides the quantitative bite that makes the direction, given by the elasticity of substitution, stronger or weaker.

To better get the intuition for this result, consider a model with only two sectors and one factor of production that we call labor. We assume labor is the numeraire. Define the Allen-Uzawa elasticity of substitution between any pair of inputs (k, h) by a given producer j (θ^j_{kh}), as

$$\theta^j_{kh} = \frac{\frac{\partial \log M_{jk}}{\partial \log P_h}}{\Omega_{jh}}. \tag{15}$$

This elasticity of substitution is a share-weighted demand elasticity since $\frac{\partial \log M_{jk}}{\partial \log P_h}$ is the *constant-output* response of demand of producer j for good k when we change the price of good h.

Consider now a technology shock to a producer n such that $d \log A_n > 0$. The change in the Domar weight of a producer i change in response to this shock is given by

$$\frac{\mathrm{d}\lambda_i}{\mathrm{d}\log A_n} = \sum_{j=1}^{2} \lambda_j \Phi_j \left(\Psi_{(i)}, \Psi_{(n)}\right)$$

$$\Phi_j \left(\Psi_{(i)}, \Psi_{(n)}\right) = -\sum_{k=1}^{2}\sum_{h=1}^{2} \Omega_{jk} \left(\delta_{kh} + \left(\theta^j_{kh} - 1\right) \Omega_{jh}\right) \Psi_{ki}\Psi_{hn}$$

$$\delta_{kh} = 1 \text{ if } k = h \quad \text{and 0 otherwise}$$

where we use $\Psi_{(i)}$ to denote the ith column of the Leontief-inverse matrix $\mathbf{\Psi}$. $\Phi_j\left(\Psi_{(i)}, \Psi_{(n)}\right)$ is what Baqaee and Farhi (2019) call the *input-output substitution operator*. This operator is important because it records how each producer j redirects expenditures towards sector i after a change in sector productivity n. To fix ideas, take a given producer j. In the presence of intermediate input linkages, an increase in the technology of sector n translates into a price change of good h of -Ψ_{hn}. Following this decrease in the price of good h, sector j may reallocate its expenditure from other goods k towards good h. How much it does so is measured by $\Omega_{jk}\left(\delta_{kh} + \left(\theta^j_{kh} - 1\right)\Omega_{jh}\right)$ that provides the partial equilibrium change in expenditure share of producer j on good k when we vary the price of good h i.e., $\frac{\partial \Omega_{jk}}{\partial \log P_h}$. This effect is then transmitted *upstream* in the production network (from the buyer to the seller) from producer k, the one that producer j was redirecting expenditure towards/from, to producer i by the element Ψ_{ki} that records how important is producer i as a seller to producer k. This chain of reasoning holds for all producers j that potentially demand good i. The final effect of producer j on producer i is weighted by the size of sector j, i.e $\lambda_j \Phi_j(\Psi_{(i)}, \Psi_{(n)})$.

As a final remark, we highlight that the previous simple example suggests that Domar weight responses depend both on the supply side and the demand side of the economy. This result means that both the roles of each firm as a supplier (outdegree) and buyer (indegree) matter for sales responses and thus shape its cross-sectional distribution. This dependence provides a rationale for using outdegrees and indegrees to explain the cross-sectional outcomes we studied in the firm-level empirical evidence.

Simple Quantitative Example. We now conduct a simple quantitative exercise to illustrate how this measure affects aggregate output skewness. Imagine a two-sector world where

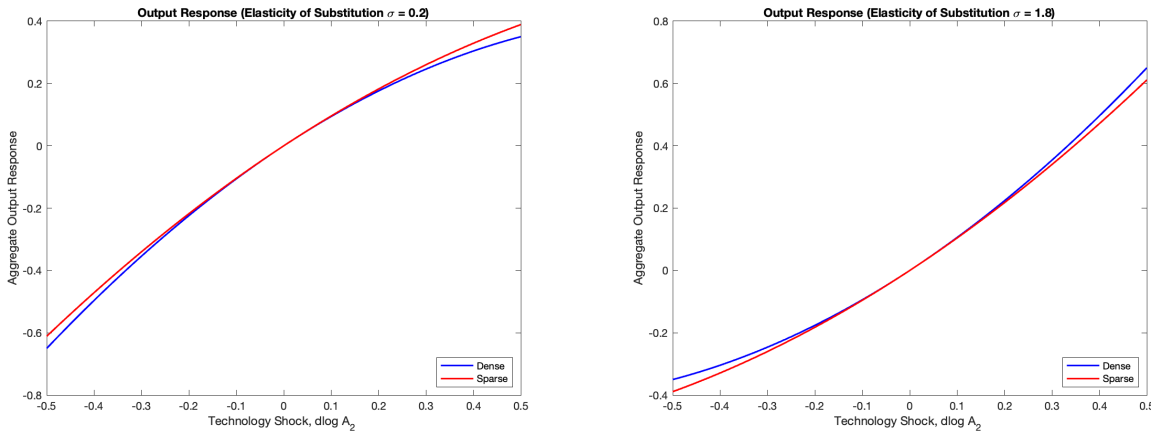
$$\mathbf{\Omega}^{sparse} = \begin{bmatrix} 0 & (1-a) \\ (1-a) & 0 \end{bmatrix} \tag{16}$$

$$\mathbf{\Omega}^{dense} = \begin{bmatrix} (1-a)/2 & (1-a)/2 \\ (1-a)/2 & (1-a)/2 \end{bmatrix}. \tag{17}$$

Figure 10 shows the aggregate output response to a technology shock in sector 2, for two different elasticities of substitution $\sigma = 0.2$ and $\sigma = 1.8$, and the two different network structures detailed in Equations (16) and (17) assuming $a = 0.5$. We construct this exercise by changing expenditure distribution on intermediate inputs while keeping the aggregate expenditure on intermediates, the consumption shares, and the Domar weights equal in both cases. If inputs are gross complements, as shown by Baqaee and Farhi (2019), aggregate

Figure 10. Aggregate Output Response to a Technology Shock in Sector 2

(a) $\sigma = 0.2$ (b) $\sigma = 1.8$



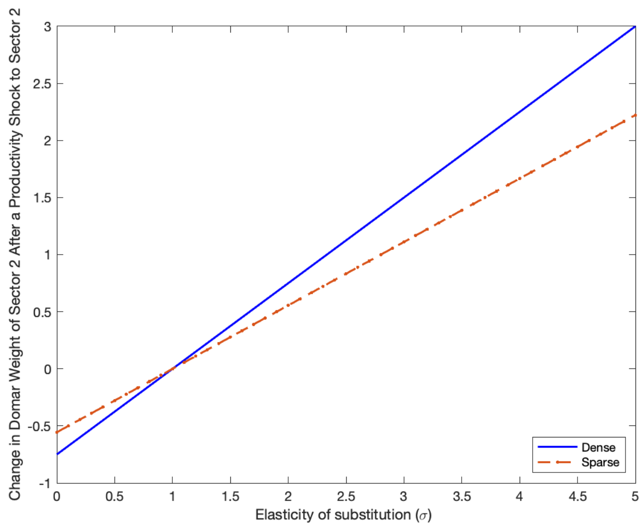
Note: This figure plots aggregate output responses to a technology shock in sector 2 for different elasticities of substitution and different network structures. The blue line shows the aggregate output responses with a dense network structure, while the red line shows the aggregate output responses with a sparse network structure.

output is a concave function of productivity, implying that negative shocks are amplified and positive shocks are attenuated. Our contribution here is to demonstrate that the dense network displays a stronger concavity of aggregate output compared to the sparse network. In panel (b), we observe that the opposite holds when inputs are substitutes in production. In that case, negative shocks are mitigated, while positive shocks are amplified. Both effects are stronger in the dense network.

Figure 11 explains the intuition behind the results in Figure 10. When inputs are gross complements, positive productivity shocks to sector 2 shrink the sector's size. The decline in the Domar weight of sector 2 is larger in the dense network compared to the sparse, which explains the stronger concavity in production observed for the more interconnected network. Exactly the opposite holds when the elasticity $\sigma > 1$. This result is akin to Baumol's cost disease. Under complementarities, the sector in which productivity declines becomes larger in the economy, which further amplifies the negative effect of the initial shock.

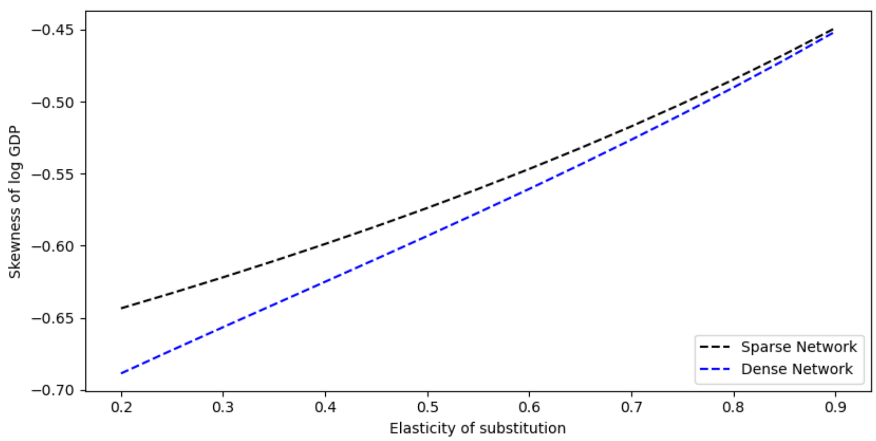
Finally, in Figure 12, we show the implications for the skewness of log real GDP in both networks. As we observe, the smaller the elasticity, the more negative the skewness of output in the dense network compared to the sparse one.

Figure 11. Changes in Domar Weight of Sector 2 after a positive technology shock



Note: This figure shows $d\lambda_2/d\log A_2$ for different values of the elasticity of substitution σ.

Figure 12. Simulated Skewness as a Function of the Elasticity



Note: This figure shows the simulated skewness of log real GDP in the dense and sparse networks, and the value of the elasticity of substitution σ in the x-axis. Productivity shocks follow a normal distribution with a mean of 1.5 and a standard deviation of 0.25. The skewness reported is the skewness of 20,000 simulations. This corresponds to the global solution of the model.

5 Quantitative Exercises

In this section, we calibrate a production network model with non-unitary elasticities of substitution between inputs and constant returns to scale in production, as in Baqaee and Farhi (2019), Miranda-Pinto (2021), and Carvalho et al. (2021). We perform two quantitative exercises. First, we use the OECD industry-to-industry production network for the 46 countries in our sample from section 2. Our goal is to show how the model can generate the observed relationship between macroeconomic skewness and network density. In our second exercise, we use the firm-to-firm network data for the Chilean economy from Section 3 to study the ability of our model to generate the cross-sectional patterns that relate firm-level networks to firm-level output asymmetry. Here, we also use the firm-to-firm calibration to evaluate the ability of our model to generate a negatively skewed distribution of real GDP.[14] A key parameter in our quantitative exercises is the elasticity of substitution between inputs. We use an elasticity of substitution between inputs σ of 0.55 from Fujiy et al. (2022) for both calibrations.[15]

5.1 Intersectoral linkages and skewness across countries

We use the OECD input-output data for the 46 countries in our sample for the year 1995 to calibrate the model input-output shares Ω_{ij}, labor shares Ω_{if}, and consumption shares b_i. We follow Baqaee and Farhi (2019) and assume that sectoral productivity A_i follows a log-normal distribution with mean $-\Sigma_{ii}/2$ and standard deviation Σ_{ii}. For simplicity, we assume $\Sigma_{ii} = 12\%$ for all i. We obtain the model-implied skewness of log real GDP using the global solution of the model and simulating 5,000 draws of productivity. Figure 13 depicts the skewness of log real GDP across countries implied by the model. Two observations are consistent with the empirical evidence in Section 2.

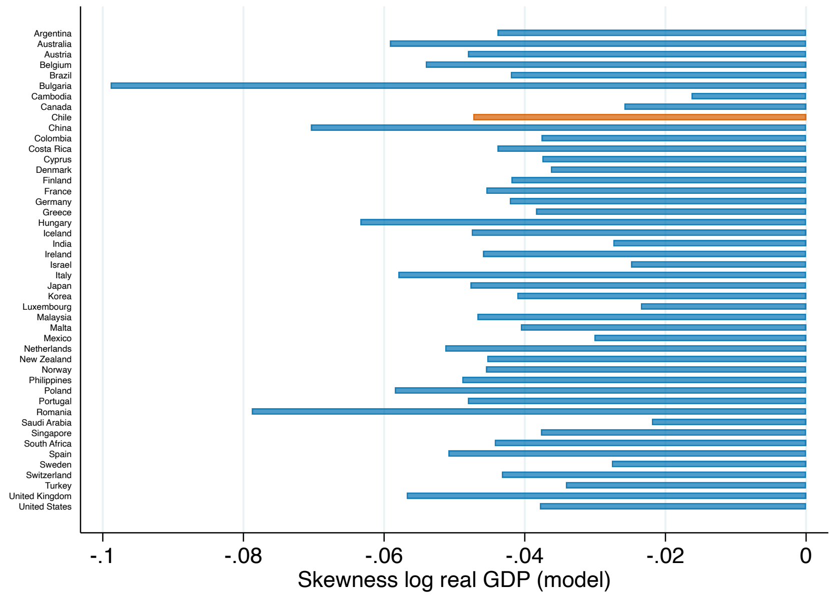
First, the model can deliver a negative skewness for almost all countries, and the implied skewness shows significant heterogeneity. However, the model falls short of replicating the level of skewness. This shortfall is not surprising given that we are not targetting the level and that our calibration assumes common volatility of productivity shocks across sectors and countries. Hence, we are not leveraging the potential heterogeneity in cross-country sectoral productivity and sectoral elasticities. Indeed, Chile's skewness in this calibration is -0.023, while the firm-to-firm network calibration in the next section can generate a skewness of -0.3, which is more in line with the observed value in Figure 1.

Second, as we observe in Figure 14, the model is able to deliver the observed negative relationship between log real GDP skewness and production network density we document in section 2, Figure 3.

[14] Note that real GDP in this model is stationary by construction and thus is the model counterpart of the cyclical component we analyze in the empirical section.

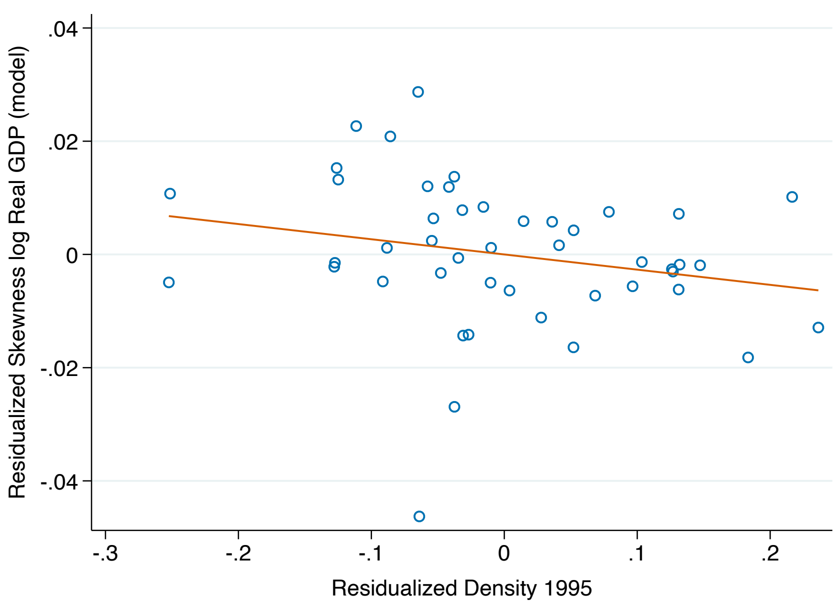
[15] The authors' estimate for the elasticity between inputs in India during COVID-19 lies within the range of the estimates from Boehm et al. (2019) and Atalay (2017). Although Miranda-Pinto (2021) and Miranda-Pinto and Young (2022) find substantial sectoral heterogeneity in production elasticities, to focus on the role of networks and complementarities, we assume homogeneous elasticities.

Figure 13. Skewness of real GDP (model)



Note: This figure plots the model implied skewness of countries' log real GDP using 5,000 draws from the model. Shocks to sectoral productivity are iid log-normal with a standard deviation of 12%.

Figure 14. Model Implied Relationship Production Network Density and Skewness



Note: This figure plots the residualized skewness of log real GDP simulated from the model using 5,000 draws and the residualized network density in 1995. These are residualized using the following controls: the skewness of the weighted outdegree and the skewness of the weighted indegree.

5.2 Firm-to-firm network in Chile

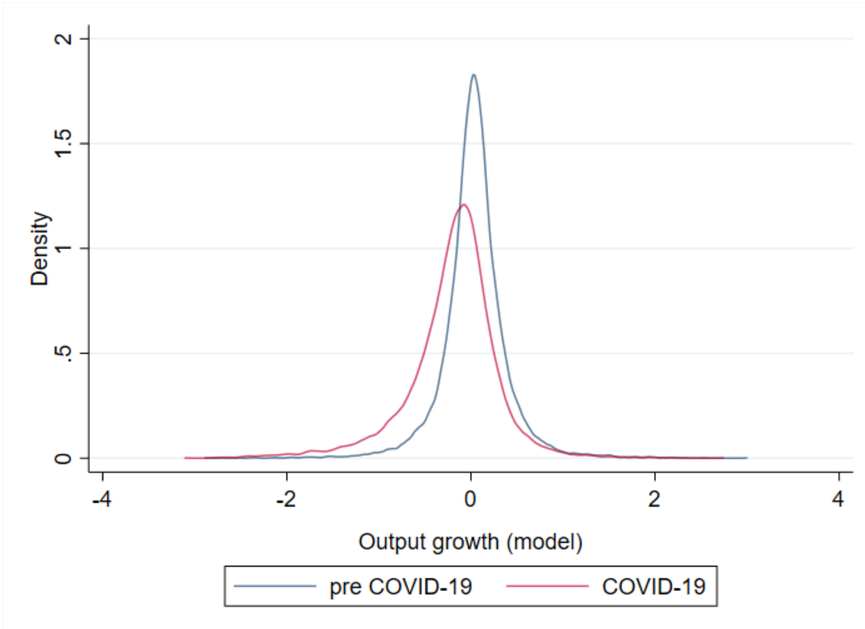
We construct our firm-to-firm production network ($\mathbf{\Omega}$) using the detailed transaction data from Chile used in section 4. We also calculate value-added input shares a (capital and labor). Our calibration assumes that the input-output shares and value-added input shares in 2019q2 describe the steady state of the economy. Hence, an element Ω_{ij} is the ratio between intermediate inputs that firm i spends on firm j's output as a fraction of firm i's total sales. The vector of value-added input shares is one minus the share of intermediate inputs in gross output. The vector of consumption shares $\boldsymbol{b}$ is assumed to be symmetric for all sectors $1/N$. We make this decision due to data limitations and to focus on the role of the production network.

Our sample of firms is substantially smaller than the sample of firms we had in previous sections. To be consistent with our model (no entry/exit and exogenous linkages), we choose the network of firms that display active linkages and positive sales throughout 2019q2-2020q2. We also keep firms with labor productivity data for at least eight quarters. Our final sample includes $N = 16,255$ firms.

We calibrate firm-level productivity using data on firm-level revenue labor productivity $LP_{it} = \frac{P_{it}Q_{it}}{E_{it}}$, where $P_{it}Q_{it}$ and E_{it} are, respectively, total sales and total employment of firm i at time t. We compute changes in productivity relative to the previous year's quarter. For example, the percent change in labor productivity during COVID-19 is $\Delta \log A_{i,2020q2} = \log LP_{i,2020q2} - \log LP_{i,2019q2}$. Therefore, we map our model to the data through the evolution of firm-level productivity. Our first goal is to investigate whether the model can generate the procyclical skewness of output growth we observe in the data. Figure 15 plots the model-implied cross-sectional distribution of output growth pre-COVID-19 (2020q1) and during COVID-19 (2020q2). The cross-sectional distribution of output growth becomes substantially more asymmetric during COVID-19. While output growth skewness in 2020q1 was 0.04, it declined to -0.77 during 2020q2. This result is not driven by the skewness of the shock to productivity but entirely due to the mechanism we highlight in the paper: large negative shocks are further amplified by complementarities in production and the network structure. Indeed, as observed in Table B.2 in the Appendix, during 2020q2, the average percent change in productivity is -0.04 (compared to 0.05 in 2020q1), and the skewness of cross-sectional productivity growth is -0.10 (compared to -0.09 in 2020q1).

We now investigate the role of firm-to-firm linkages in amplifying the decline in output growth during COVID-19. To do so, we study the ability of the model to generate the empirical pattern between log degrees and output growth previously documented in Figure

Figure 15. Model implied distribution of output growth distribution



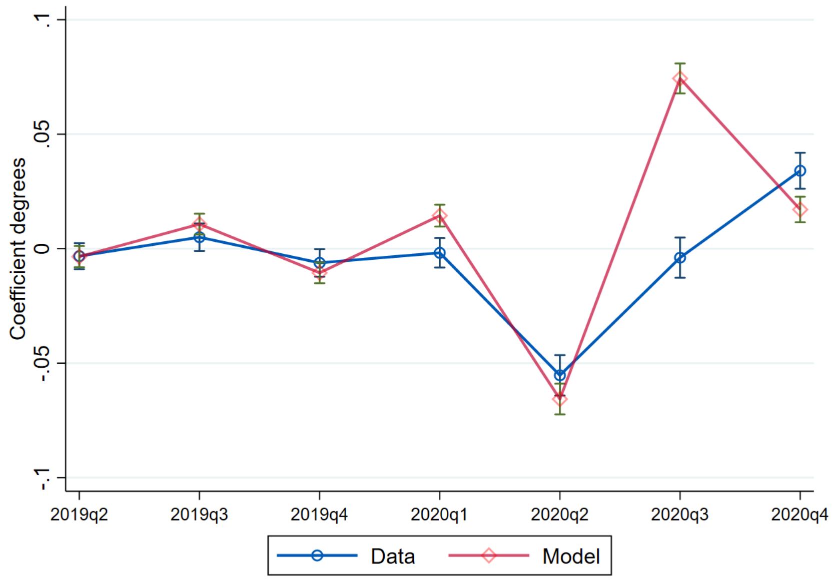
Note: This figure plots the kernel density of output growth from the model calibrated to 2020q1 (pre-COVID-19) and 2020q2 (COVID-19). Each quarter in the model corresponds to a different level of firm-level productivity, calibrated from the data.

9 from estimating Equation 3. Figure 16 plots the model implied coefficient $\beta(t)$ in Equation 3 for different quarters in 2019 and 2020. We also plot in the same figure the coefficients and confidence intervals of the empirical estimates of $\beta(t)$. Our results show a pattern similar to that in the data. More interconnected firms saw larger declines in output during COVID-19 and also recovered faster. During other quarters, when the size of the shocks is smaller, the relationship between output growth and degrees vanishes, which emphasizes the role of non-linearities in the model.

Finally, we study the ability of the model to generate aggregate skewness. To do so, we use the time series of firm-level productivity to measure firm-level volatility of shocks. We follow Baqaee and Farhi (2019) and assume that firm-level productivity is iid and log-normally distributed, with mean $-\varsigma_i/2$ and standard deviation ς_i. We measure the skewness of log real GDP from $S = 10$ simulations of $T = 100$ periods each.[16] The average skewness over the simulations is -0.31, which is significantly larger than the one implied by the industry-to-industry calibration (-0.047) for Chile and much closer to that in Figure 1.

[16]The solution of the model entails inverting a square matrix of dimension $N \approx 17,000$, implying that each simulation takes a considerable amount of time.

Figure 16. Coefficient of regressing output growth against log degrees



Note: This figure plots the OLS estimated $\beta(t)$ from $d\log q_i(t) = \alpha + \beta(t)\log degree_i + \gamma(t)\log sales_i + \epsilon_i(t)$, using the model implied $d\log q_i$. Each quarter in the model corresponds to a different level of firm-level productivity, calibrated from the data.

6 Conclusion

We showed that denser networks are related to business cycle asymmetries in the data – across countries, sectors, or firms, entities with more connections experience more negatively-skewed distributions of economic output. In our model, this correlation is driven by the concavity of aggregate output with respect to productivity, which complements existing results on the sources of skewed business cycles (such as financial constraints) and highlights the importance of solving models globally.

While our model is efficient, our results have policy implications that we intend to explore in future work. For example, frictions at the sectoral level can amplify sectoral shocks inefficiently, as in Bigio and La'O (2020) and Miranda-Pinto and Young (2022). The gains of industrial policies in such environments that reallocate inputs across sectors will hinge on how connected the network is (Liu, 2019), and therefore also whether that network structure amplifies or dampens negative shocks. Since negative skewness imposes larger costs of fluctuations on households, understanding its source is important for assessing the welfare costs of cycles as well.

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Appendix

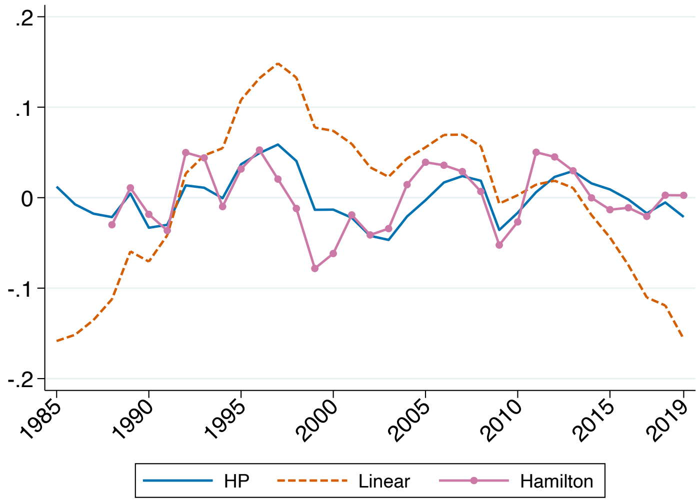
A Cross-Country Evidence: Real GDP Detrending

Here, we compare the performance of the Hamilton filter against two commonly used detrending procedures: Hodrick-Prescott Filter and linear detrending. We use a smoothing parameter equal to 100 for the Hodrick-Prescott filter.

Figure A.1 shows the cyclical component for the Chilean real GDP under the three different scenarios. We note that the Hamilton filter performs better than the other two procedures for our purposes as it better captures crises. For example, the Hamilton filter (pink line) captures the decline in economic activity around 1998 due to the Asian Financial Crisis that badly hit the Chilean economy, among other emerging markets economies, while the other two detrending procedures view the crisis as merely a decline towards the mean.

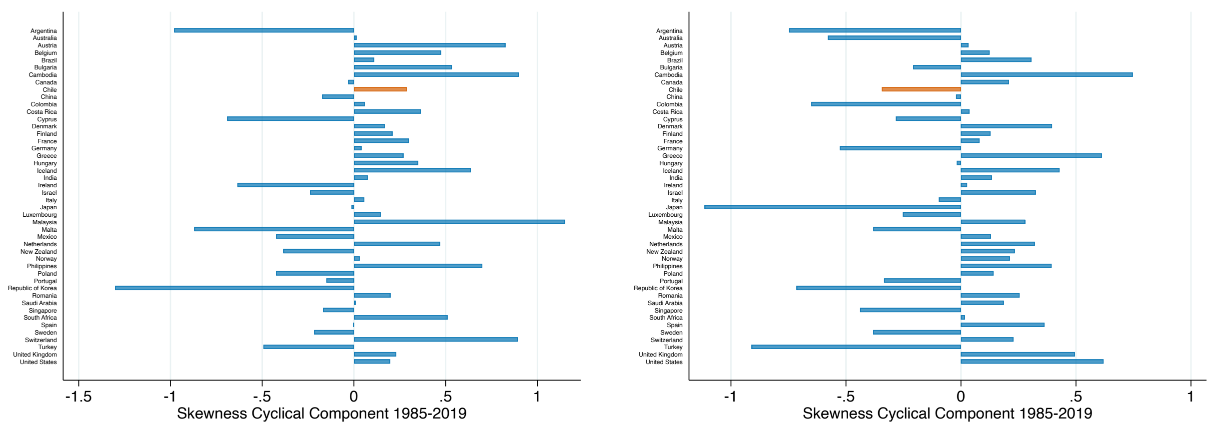
The fact that the Hamilton filter better captures crises is not particular to the Chilean economic data but holds more broadly in the cross-section. In Figure A.2, we plot an analog to Figure 1 — where we used the Hamilton filter — but using the other two methods. As it is clear from the figure, both methods produce inconsistent results, generating negative skewness of the cyclical component of real GDP only on around half of the countries in our sample. In contrast, the Hamilton filter captures the skewed business cycles characteristic of real GDP.

Figure A.1. Chilean Real GDP Cyclical Components under different detrending procedures



Note: The figures shows the cyclical component of real GDP for Chile between 1985 – 2019 using three different sets of detrending procedures. The solid blue line uses a Hodrick-Prescott filter with a smooth parameter equal to 100. The orange dashed line shows the cyclical component when using linear detrending. Finally, the dot-solid pink line shows the cyclical component using the Hamilton filter.

Figure A.2. Cross-Country Skewness of Real GDP Cyclical Components



(a) Hodrick-Prescott Filter

(b) Linear Detrending

Note: Panel (a) shows the skewness of the cyclical component of (log) real GDP when using the Hodrick-Prescott filter with a smooth parameter equal to 100. Panel (b) shows the skewness of (log) real GDP using a linear detrending procedure.

B Additional Tables

Table B.1. Cross-Country Relationship between Real GDP Cyclical Component Skewness and Network Density

	(1)	(2)
Density 1995	-2.766***	-2.977***
	(0.912)	(1.074)
Obs.	46	46
R^2	0.220	0.260
Controls	No	Yes

Note: This table reports the OLS coefficient of running a regression between real GDP cyclical component skewness as the dependent variable. Column 1 uses network density in 1995 as an independent variable. Column 2 uses the following controls: GDP per capita in 1985, the volatility of the cyclical component of real GDP, and the skewness of the outdegree and indegree distribution in 1995.

Table B.2. Annual labor productivity growth

	2019q2	2019q3	2019q4	2020q1	2020q2	2020q3	2020q4
Mean	0.01	0.04	0.02	0.05	-0.04	0.07	0.11
Median	0.02	0.04	0.02	0.05	-0.03	0.07	0.11
St. Dev	0.27	0.27	0.27	0.28	0.35	0.35	0.32
Skewness	-0.04	-0.09	-0.06	-0.09	-0.10	-0.18	-0.15
Observations	16,938	16,938	16,938	16,938	16,938	16,938	16,938

Note: This table presents the descriptive statistics of annual labor revenue labor productivity growth at different quarters. The sample of firms corresponds to that used in Section 5.2 for our firm-to-firm network calibration.